*Below is the unedited draft of the article that has been accepted for publication*
*(© Physics of Life Reviews, 2017, V. 21, P. 1-20.)*

# Topodynamics of metastable brains

**Arturo Tozzi[1], James F. Peters[2], Andrew A. Fingelkurts[3], Alexander A. Fingelkurts[3], Pedro C. Marijuán[4]**

*[1] Center for Nonlinear Science, University of North Texas, Denton, USA*
*[2] Department of Electrical and Computer Engineering, University of Manitoba, Canada and Department of Mathematics, Adıyaman University, Adıyaman, Turkey*
*[3] BM-Science – Brain and Mind Technologies Research Centre, Espoo, Finland*
*[4] Bioinformation Group, Aragon Institute of Health Science (IACS), Aragon Health Research Institute (IIS Aragon) Zaragoza, Spain*

## Abstract

The brain displays both the anatomical features of a vast amount of interconnected topological mappings as well as the functional features of a nonlinear, metastable system at the edge of chaos, equipped with a phase space where mental random walks tend towards lower energetic basins. Nevertheless, with the exception of some advanced neuro-anatomic descriptions and present-day connectomic research, very few studies have been addressing the topological path of a brain embedded or embodied in its external and internal environment. Herein, by using new formal tools derived from algebraic topology, we provide an account of the metastable brain, based on the neuro-scientific model of Operational Architectonics of brain-mind functioning. We introduce a "topodynamic" description that shows how the relationships among the countless intertwined spatio-temporal levels of brain functioning can be assessed in terms of projections and mappings that take place on abstract structures, equipped with different dimensions, curvatures and energetic constraints. Such a topodynamical approach, apart from providing a biologically plausible model of brain function that can be operationalized, is also able to tackle the issue of a long-standing dichotomy: it throws indeed a bridge between the subjective, immediate datum of the naïve complex of sensations and mentations and the objective, quantitative, data extracted from experimental neuro-scientific procedures. Importantly, it opens the door to a series of new predictions and future directions of advancement for neuroscientific research.

**Keywords**:
Topology; Borsuk-Ulam theorem; nonlinear dynamics; central nervous system; mind.

**Abbreviations**:
BUT: Borsuk-Ulam theorem.
FPT: fixed point theorem.
IPST: internal physical space–time.
OA: operational Architectonics.
OM: operational module.
OST: operational space–time.
PST: phenomenal space–time.
RTP: rapid transitive period.

## INTRODUCTION

Over the last several decades, there has been an explosive development of new theories regarding brain and mind functioning as well as new powerful techniques that facilitate innovative experimental studies (see for example, [1-3]. However, none of these theories exploits in an *explicit* way the actual physical (brain) and subjective (mind) *operations* the human brain-mind performs in the course of behavior. The recently proposed Operational Architectonics (OA) framework of brain-mind functioning [4-10], by contrast to other theories, sets out with the explicit aim to describe, measure and model the brain and mind operations involved in the development of the most complex human behaviors, as governed by the brain. It is self-evident that both brain and mind functioning have intertwined temporal structure defined by their respective operations [11-13], where metastable changes of brain spatiotemporal patterns are isomorphic with cognitive and phenomenal levels [8,10].

This paper aims to throw a bridge between the above-mentioned OA model and the far-flung branch of algebraic topology. Indeed, topology, in assessing the properties that are preserved through deformation, stretching and twisting of objects, is an underrated methodological approach with countless possible applications [14-18]. In particular, the Borsuk-Ulam theorem (BUT), cast in a quantitative fashion which has the potential of being operationalized, stands for a universal principle underlying a number of natural phenomena [19]. BUT states that, if a single point on a circumference projects to a higher spatial dimension, it gives rise to two antipodal points with matching descriptions, and vice versa (**Figure 1A**) [20-23]. Points on a sphere are *antipodal*, provided they are diametrically opposite [24], such as, for example, the poles of a sphere. This means, *e.g.,* that there exist on the earth's surface at least two antipodal points with the same temperature and pressure. BUT looks like a translucent glass sphere between a light source and our eyes: we watch two lights on the sphere surface instead of one. But the two lights are not just imagined, they are also real with observable properties, such as intensity and diameter. This means that two antipodal points can be described at one level of observation, while relative to just a single point at a lower level [18,25]. Here we will show that BUT and its recently developed variants allow the assessment of brain functioning in terms of affinities and projections from real spaces to abstract ones.

This paper comprises four sections. The first section introduces the model of Operational Architectonics that considers the brain as a metastable system, thus placing the brain in the broader framework of dynamical systems theory. The second section illustrates, in plain terms accessible for a broader audience, the topological apparatus able to quantitatively describe the activity and function of the metastable brain. In particular, we will go through novel variants of the above mentioned BUT. The third, crucial, section throws a bridge between the two first parts, assessing the activities of the brain operational architectonics in terms of algebraic topology. We termed this novel approach "*brain topodynamics*". In the fourth section, we mention several predictions and new research directions that follow from the OA/topological framework. This section aims also to answer the most important question: in the context of OA, what does such topodynamical approach bring to the table, with respect to both conceptual and operational points of view? What new predictions can be established and what new research directions might be followed?

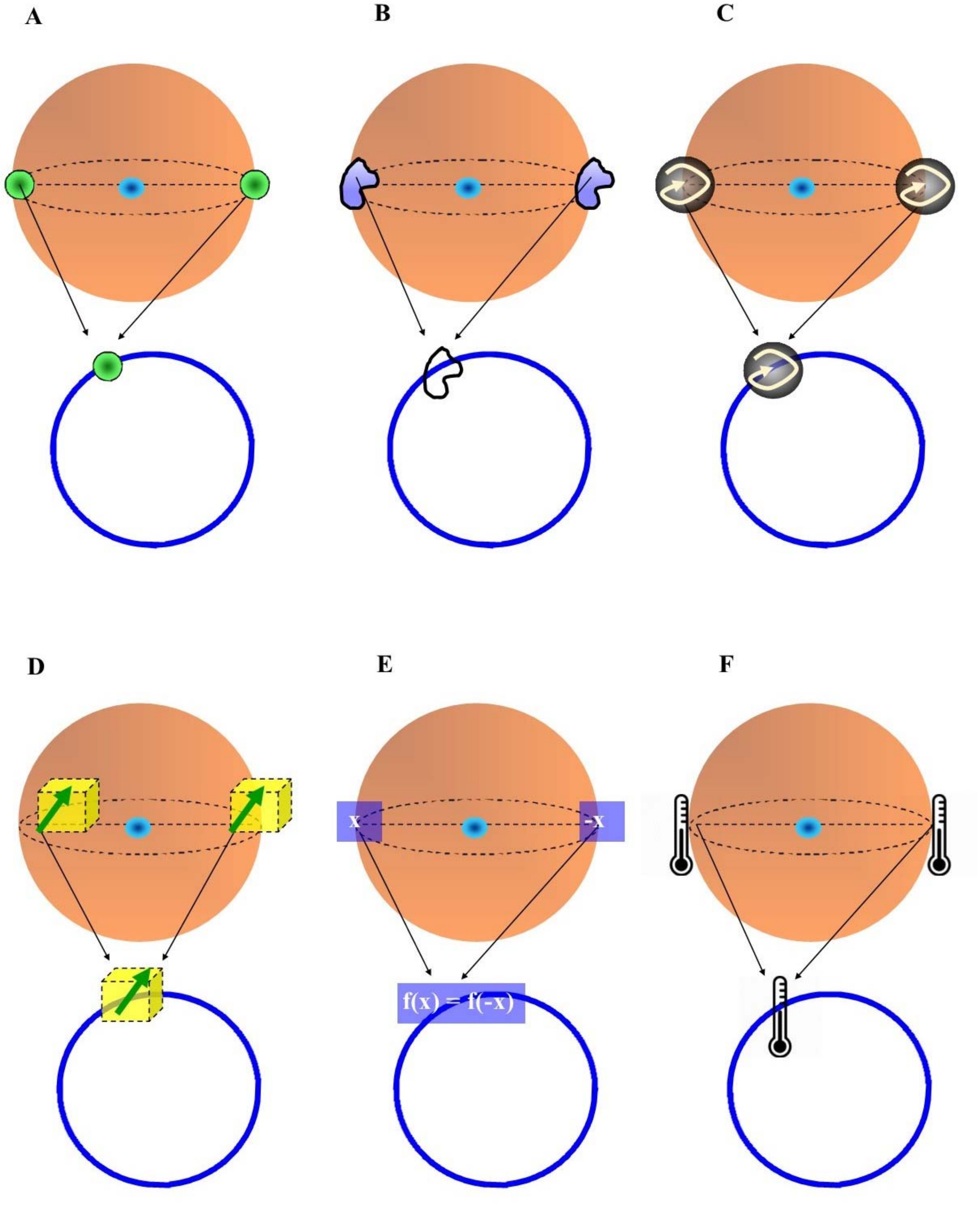

**Figure 1. Possible types of antipodal features in Borsuk-Ulam theorem (BUT).** The original formulation of BUT, with two antipodal points, is illustrated in **Figure 1A**. **Figures 1B-1F** display, respectively, two antipodal shapes, trajectories, vectors, functions and temperatures. Each set of antipodal features on a sphere stands for a single feature on a circumference, if we evaluate it just one dimension lower.

## 1. BRAIN METASTABILITY AND BRAIN-MIND OPERATIONAL ARCHITECTONICS MODEL

Recently a novel paradigm in relation to the brain-mind functioning is beginning to emerge—it stresses the dynamic balance between isolated functions of local neuronal assemblies and globally coordinated activity among them [8,12,13]. In this new view, the potential multivariability of the neuronal assemblies and their larger conglomerates (networks) appears to be far from continuous in time, but confined by the dynamics of short-term local and global *metastable* brain states [26]. As it has been proposed, metastability is an entirely new conception of brain functioning, where the individual parts of the brain exhibit tendencies to function autonomously at the same time as they exhibit tendencies for coordinated activity [11, 26-28]. Specifically, *metastability* (when the system's degrees of freedom are restricted) is circumstantial for the interaction among the elementary neuronal systems (neuronal assemblies) in order to generate adaptive behavior within changing and not fully predictable environments: "By synchronizing the stable microstates of the 'microscopic variables' during certain periods, the neuronal systems (neuronal assemblies) have the possibility for interactive information exchange of the essential variables, which are important for the acceptance and expression of 'consensual decision' that is appropriate for the functional requirements engendered by each successive stage of behavioral/cognitive/mental performance" (p. 851 in Ref. [26]). In the metastable regime of brain functioning [26,28], the interplay of these two tendencies (autonomy and integration) represents the coexistence of a complementary and not antagonistic pair [29, 30]. Thus, metastability in the brain refers to competition of complementary tendencies of cooperative integration and autonomous fragmentation among many distributed neuronal assemblies [11, 26, 28, 31]. Importantly, metastability explicitly refers to transient, nonstationary processes; and in this respect it differs from synergetics [32], where the main principle is to compress the effective number of degrees of freedom in complex systems to a few "order parameters" or variables that adequately approximate system dynamics at large scales [33].

The metastability phenomenon in the brain is most clearly expressed and best observed at the level of an *electromagnetic field* which is highly nested, self-organized and structured in temporal and spatial domains [4, 7, 9, 26]. Such a brain field is manifested as the interplay of multiple local oscillatory waves accessible through local electroencephalographic (EEG) measurements [7, 34-36]. In that regard: What could be a model of brain-mind functioning especially well suited for studies of the nonstationary structure of the EEG field and which is uniquely capable of assessing the dynamic and metastable changes of brain spatial–temporal patterns that are isomorphic with cognitive and phenomenal levels [8, 10]? Essentially this model should take repetitions of spatial–temporal patterns of brain field into account at all functional levels, thus capturing both dynamic as well as hierarchical complexities of brain activity nested within a multi-scale operational architecture [9]. In a series of publications, Fingelkurts and Fingelkurts [4-7] have proposed the Operational Architectonics (OA) framework of brain-mind functioning centered around the notion of *operation* that satisfies these requirements. The notion of 'operation' has been chosen as the center of this

OA theory of brain-mind functioning due to several reasons: it is a process lasting in time, present in both brain and mind functioning, and it has combinatorial nature (increasing complexity). All these features make ‘operation’ especially well suited for understanding and studying the mechanisms of how a conscious mind emerges from the physical brain (for a more detailed discussion see [8-10].

According to a general theory of brain-mind OA [4-10], the simplest mental/cognitive operations (responsible for elemental qualia or simple computations) are presented in the brain in the form of local 3D-fields produced by transient functional neuronal assemblies, while complex mental/cognitive operations (responsible for complex objects, images or thoughts) are brought into existence by joining a number of simple operations (temporal coupling of local 3D-fields by means of *operational synchrony*, OS) in the form of so-called *operational modules* (OM) of varied complexity and life-span. Therefore, brain OA is presented as a highly structured and dynamic extracellular electric field nested in spatial and temporal domains and spanning over a range of frequencies, thus forming a particular *operational space–time* (OST) in the brain [9] (**Figure 2**). What is curious about the OST level of brain organization is that it intervenes between brain *internal physical space–time* (IPST) level where it literally resides, and the experiential/subjective phenomenal structure of a mind (*phenomenal space–time*, PST) to which it is isomorphic [9]. In this way, the OST level (i) has emergent properties relatively independent from the neurophysiological/neuroanatomical properties of the IPST level, and (ii) has a one-to-one correspondence with phenomenal (PST) level that supervenes on, and is ontologically inseparable from, the operational (OST) level (**Figure 2**). The OST exists within brain internal physical space–time (IPST) and is best captured by the EEG measurement [4, 6, 7, 9, 10].

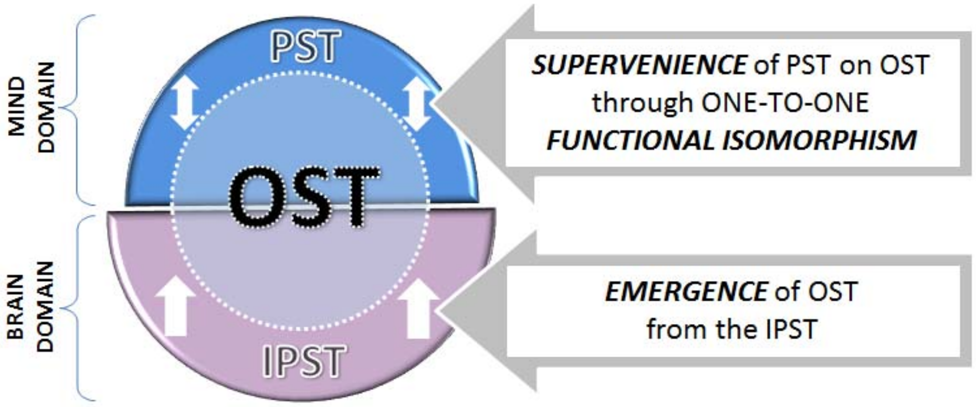

**Figure 2.** Phenomenal, operational and physical levels of the brain–mind organization and their relation to emergentism and supervenience. Electromagnetic brain field constituting the OST level is the emergent property of brain itself (IPST level). The phenomenal level (PST) supervenes on the operational level (OST) and is functionally isomorphic to it. IPST indicates the *internal physical space–time* of the brain (violet color); OST indicates the *operational space–time* of the brain (indicated by white puncture line); PST indicates the *phenomenal space–time* of consciousness (blue color). In this model the OST level represents the constitutive mechanism of phenomenal consciousness; it ties the phenomenal (subjective) and neurophysiological (physical) levels together. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

At the EEG level, simple mental operations (phenomenal qualities, primary cognitive operations and emotions) are equivalent to the EEG quasi-stationary segments (within which the local fields generated by transient functional neuronal assemblies are expressed) [4, 8, 9]. The quasi-stationary EEG segments within each local EEG signal are marked by boundaries in the form of *rapid transitional processes/periods* (RTPs) (Fig. 3, upper panel), which could be observed within a short-time window, within which EEG amplitude changes abruptly [6]. In comparison to the length of quasi-stationary segments, each RTP has a very short duration and can therefore be treated as a point or near-point [4, 6]. The transition from one segment to another, then, reflects the moment of abrupt switching from one neuronal assembly's operation to another (**Figure 3**, upper panel). Physically one could interpret such a transition as the offloading of entropy and the emergence of a new structure of the system (new neuronal assembly) [10]; see also [37, 38]. In the physics literature, the events similar to RTPs are referred to as renewal (or critical) events [39, 40]: namely, the events that reset the memory of the system [41]. This property is in fact a mathematical definition of a well-known physical phenomenon of "intermittency" and is compatible with self-organized criticality found both in physical systems [42, 43] and in the brain [44-49]. Recently, it has been documented that RTPs in EEG are indeed "crucial events" [50, 51].

Since the beginning and end of discrete operations performed by local neuronal assemblies are marked by sharp changes (RTPs) in the amplitude of local EEG signals, the simultaneous occurrence of such RTPs from different local EEG signals within the multichannel EEG recording could provide evidence of synchronization of simple operations performed by neuronal assemblies (located in different brain areas) that participate in the same functional act as a group (**Figure 3**, middle panel), *e.g.*, executing a particular complex operation responsible for a subjective presentation of complex objects, scenes, concepts or thoughts [4, 8-10]. Thus, the complex mental operations are reflected at the EEG level in the transitory spatiotemporal patterns formed by synchronized quasi-stationary segments separated by the RTPs, giving rise then to a completely new level of brain abstractness – *metastable brain states* [26] – which have been called *operational modules* (OM) [4, 6, 52] (**Figure 3**, bottom panel). The OMs are metastable because of intrinsic differences in the activity between neuronal assemblies, which constitute every OM, each doing its own job while at the same time still retaining a tendency to be coordinated together within the same OM in order to execute the macro-operation [9].

Such a structure of the OM suggests that both *parallel* and *serial* processing may be just two sides of the same single underlying mechanism—synchrony of operations performed by neuronal assemblies. Parallel processing is executed by individual neuronal assemblies, while serial processing emerges as a result of the formation of OMs and their changes along with shifts in the process of actualization of objects in the physical or the mental world [5]. In this way, computationally, OMs have both classical, connectionist, and process architectures. For example, they resemble connectionist networks [53], since they may serve as associative, content-addressable memories, and they are distributed across many neural assemblies. Yet, the specific spatial–temporal patterns (OMs) per se are unitary, like symbols in classical logics [54]. And yet, each OM is a process, since it lasts as long as several operations that have particular continuity in time (and which are produced by different neuronal assemblies) are synchronized among each other during a given temporal interval [9].

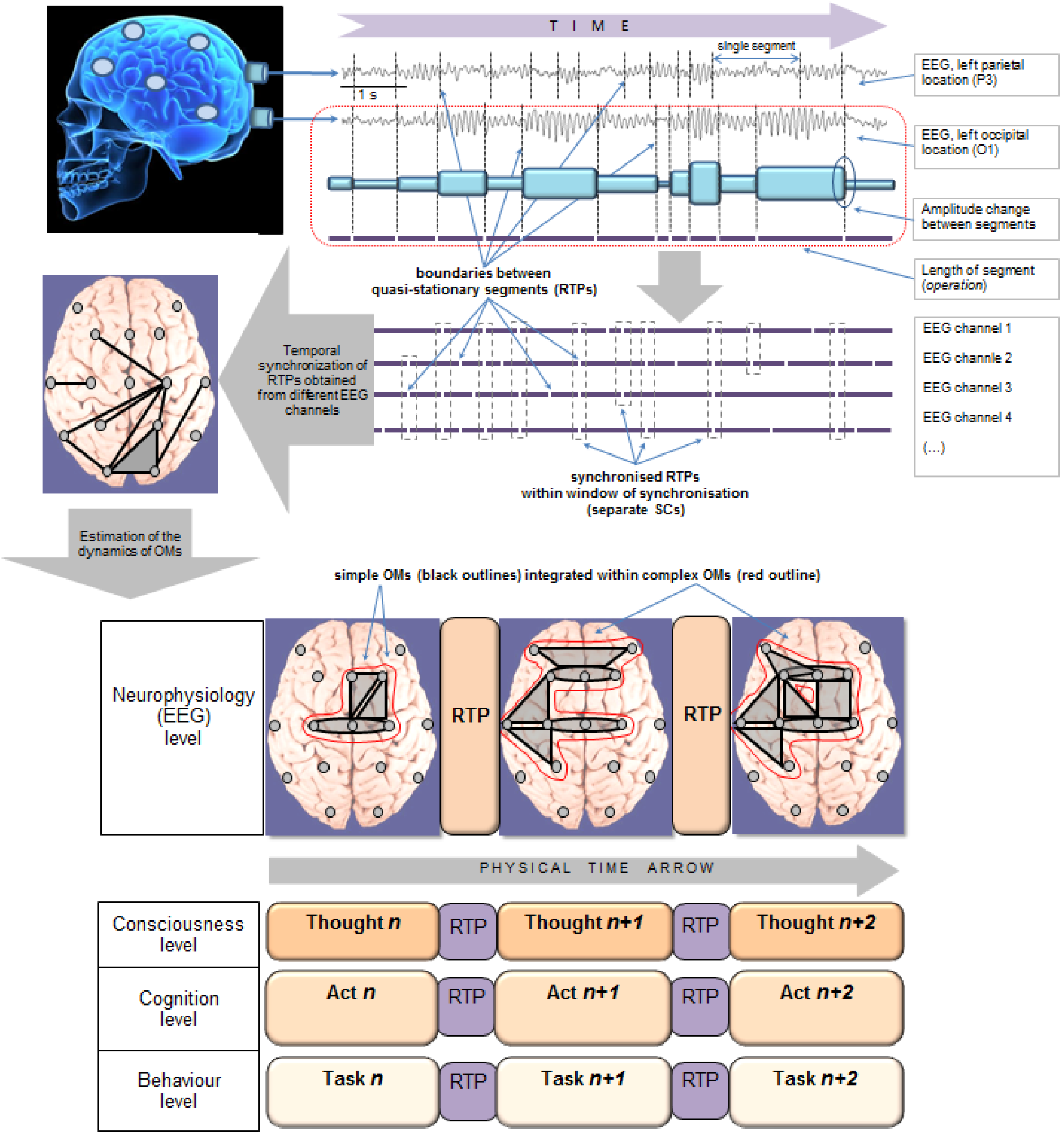

**Figure 3. Schematic illustration of EEG assessment of (a) neuronal assembly's dynamics and relation of this dynamics to simple operations and (b) large-scale conglomerates of synchronized neuronal assemblies in the form of operational modules (OMs) and their relation to a stream of complex operations or cognitive/conscious acts.** Explanations are in the text. RTP – rapid transitional period (boundary between quasi-stationary EEG segments within the same local EEG signal); SC – synchrocomplex – momentary synchronization of RTPs among several local EEG signals within short temporal window of synchronization; Gray shapes illustrate individual OMs; Red line illustrates complex OMs. Low panel represents schematically the functional structures of phenomenological consciousness, cognition and behavior that functionally isomorphic with the structure of electrophysiological level. As an example, the simplest case is shown, when cognitive, phenomenal and behavioral operations/acts coincide in time (in most cases these relations are more complex). Cognitive, phenomenological, and behavioral levels illustrate the ever-changing stream of cognitive/phenomenal/behavioral acts, where each momentarily stable pattern is a particular cognitive/phenomenal/behavioral macro-operation (thought/image/act) separated by transitive fringes (or Rapid Transitional Periods; RTPs).

In the OA model, the stream of phenomenological consciousness [55] expressed as the succession of complex cognitive operations, phenomenal images or thoughts is presented by the succession of discrete and relatively stable OMs, which are separated by rapid transitive periods (RTPs), i.e. abrupt changes of OMs (Fig. 3, bottom panel). Empirical evidence indicates [56-61] that at the critical point of transition in a mental state, *e.g.*, during changes from one mental task/thought to another, the OM indeed undergoes a profound and rapid reconfiguration. In terms of the critical theory of physics [62-65] within the RTP between two consequent Oms, there is a biphasic process [10]: a brief episode when the drastic and abrupt increase in degrees of freedom among participating neuronal assemblies is accompanied by a sudden increase in entropy, information level and dimensionality, followed by a quick episode of reduction in the degrees of freedom of neuronal assemblies and rapid decrease in entropy, information level and dimensionality when the self-organization of a new presentational state emerges in the form of a new OM within the brain OST [9, 10].

## 2. THE TOPOLOGICAL APPROACH: VARIANTS OF THE BORSUK-ULAM THEOREM

As stated in the Introduction, an advanced topological interpretation of the OA model might be based on new extensions of the Borsuk-Ulam Theorem (BUT). The concept of antipodal points (see **Figure 1**) can be generalized to countless types of system signals, by introducing novel BUT variants that are region-based. A pair of opposite points can be used not just for the description of simple topological points, but also of lines, or perimeters, areas (**Figure 1B**), regions, spatial patterns, images, temporal patterns, movements, paths, particle trajectories (**Figure 1C**) [66-69], vectors (**Figure 1D**), tensors, functions (**Figure 1E**), algorithms, parameters, groups, range of data, symbols, signs, thermodynamic parameters (**Figure 1F**), or, in general, *signals*. If we simply evaluate general nervous activity instead of ‘signals’ BUT leads naturally to the possibility of a region-based, not simply point-based, brain geometry. For instance, a brain region assessed through fMRI can have features such as area, diameter, average signal value, entropy and so on. One is thus allowed to describe brain functional and/or anatomical features in terms of antipodal points on a sphere [18, 70]. It is noteworthy that BUT allows also the evaluation of the energetic nervous requirements. Indeed, there exists a physical link between the two spheres of different dimensions and their energetic features [69]. When two antipodal functions on a higher–dimensional structure project to a lower-dimensional structure, a single function is achieved [71, 27]. This means that the single mapping function on the lower-dimensional structure displays values of energy parameters lower than the sum of two corresponding antipodal functions on the higher-dimensional structure (**Figure 1F**). Therefore, in a metastable brain formed by structures with different dimensions, a decrease in dimensions gives rise to a decrease in energy. In such a way, a metastable brain/mind is achieved, in which the energetic changes do not depend anymore on thermodynamic parameters, but rather on affine connections, homotopies and continuous functions. An example is provided by a recent paper, where BUT allows the detection of Bayesian Kullback-Leibler divergence during unsure perception [71].

Descriptively similar points and regions do not need necessarily to be opposite (antipodal), or embedded on the same structure (**Figures 4A** and **4B)** [18,19]. Thereafter, the applications of BUT can be generalized also for non-antipodal neighboring points (and/or regions) on a sphere. In effect, it is possible to evaluate matching signals, even if they are not exactly "opposite" each other. As a result, the antipodal point's restriction from the "standard" BUT is no longer needed, and one can also consider regions that are either adjacent or far apart. This BUT variant applies, provided there is a pair of regions on the sphere with the same feature value. We are thus allowed to say that the two points (or regions, or whatsoever) do not need necessarily to be antipodal, in order to be labeled together [18]. In brain terms, this means, for example, that two regions on the cortical surface with the same entropy values can be assessed together.

The original formulation of BUT describes the presence of antipodal points on spatial manifolds in every dimension, provided the manifold is a convex, positive-curvature structure (*i.e*, a ball). However, many brain functions are believed to occur on functional hyperbolic manifolds in the guise of a saddle, *i.e.*, equipped with negative-curvature and concave shape. Therefore, it makes sense to look for antipodal points also on structures equipped with curvatures other than the convex one (**Figure 4C**) [73]. Whether a system structure displays a concave, convex or flat appearance, does not matter: we may always find the points with matching descriptions predicted by BUT [71]. A single, planar description can be projected to higher dimensional donut-like structures (**Figure 4D**), so that a torus stands for the most general structure which permits the description of matching points. For further details, see [71,74].

Although BUT has been originally described just in case of *n* being a natural number which expresses a structure embedded in a spatial dimension, nevertheless *n* can also stand for other types of numbers, when assessing the brain sphere. BUT can be used not just for the description of "spatial" dimensions equipped with natural numbers, but also for antipodal regions on brain spheres equipped with other kinds of dimensions, such as a temporal or a fractal one. This means, *e.g.*, that a spherical structure can be made not just of space, but also of time. The dimension *n* might stand not just for a natural number, but also for an integer, a rational, an irrational or an imaginary one [71]. For example, *n* may stand for a fractal dimension, which is generally expressed by a rational number. This makes it possible for us to use the *n* parameter as a versatile tool for the description of systems' features.

Furthermore, matching points (or regions) might project to lower dimensions on the same structure (**Figure 5A**). A sphere may map on itself: the projection of two antipodal points to a single point into a dimension lower can be internal to the same sphere. In this case, matching descriptions are assessed at one dimension of observation, while single descriptions come into play at a lower one, and vice versa. Such correlations are based on mappings, *e.g.*, projections from one dimension to another. In many applications (for example, in fractal systems), we do not need the Euclidean space (the ball) at all: a system may display an intrinsic, *internal* point of view, and does not need to lie in any dimensional space [70]. Therefore, we may think that such a system just does exist by – and on – itself.

The BUT approach emphasizes the foremost role of symmetries in the evaluation of a metastable brain. Symmetries are real invariances that underline the world and occur at every level of systems' organization [74, 75]. Bilinear forms of geometrical structures manifest symmetrical properties [76]. In other words, affine structures preserve parallel relationships. Symmetries are general structures of mathematical, physical and biological entities and provide a very broad approach, explaining also how network communities integrate or segregate information. It must be emphasized that symmetries are widespread and may be regarded as the most general feature of systems, perhaps more general than free-energy and entropy constraints too. Indeed, recent data suggest that thermodynamic requirements have close relationships with symmetries. The interesting observation that entropy production is strictly correlated with symmetry breaking in quasi-static processes paves the way to use system invariances for the estimation of both the free energy of metastable states, and the energy requirements of computations and information processing [77]. Thus, giving insights into symmetries provides a very general approach to every kind of systems function. A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level. A symmetry break is detectable at a lower dimension of observation (**Figure 5B**).

Thereafter, we can state that single descriptions are broken (or hidden) symmetries, while matching descriptions are restored symmetries. In other words, a symmetry can be hidden at the lower dimension and restored when going one dimension higher. If we assess just single descriptions, we cannot see their matching descriptions: when we evaluate instead systems one dimension higher, we are able to see their hidden symmetries. This also means, that, going from a lower to a higher level of assessment, we find more information [74]: indeed, to make an example, a three-dimensional image encompasses more information than a two-dimensional one. In sum, symmetries, single and matching descriptions stand for the common language able to describe the metastable brain: BUT and its ingredients can be modified in different guises, in order to assess a wide range of brain functions. Although this field is nearly novel and still in progress, such a methodological approach has been already proved useful in the evaluation of brain symmetries, which allow us to assess the relationships, affinities and shape-deformations among BOLD activated areas [71]. Additionally, BUT has been used in the evaluation of cortical histological images previously treated with Voronoï tessellation [78, 79]. The utilization of BUT presented here (see **Section 3**) upon the most comprehensive and neurobiologically plausible model of brain-mind functioning, e.g, the OA, shows yet another useful application of topological approaches in neuroscience.

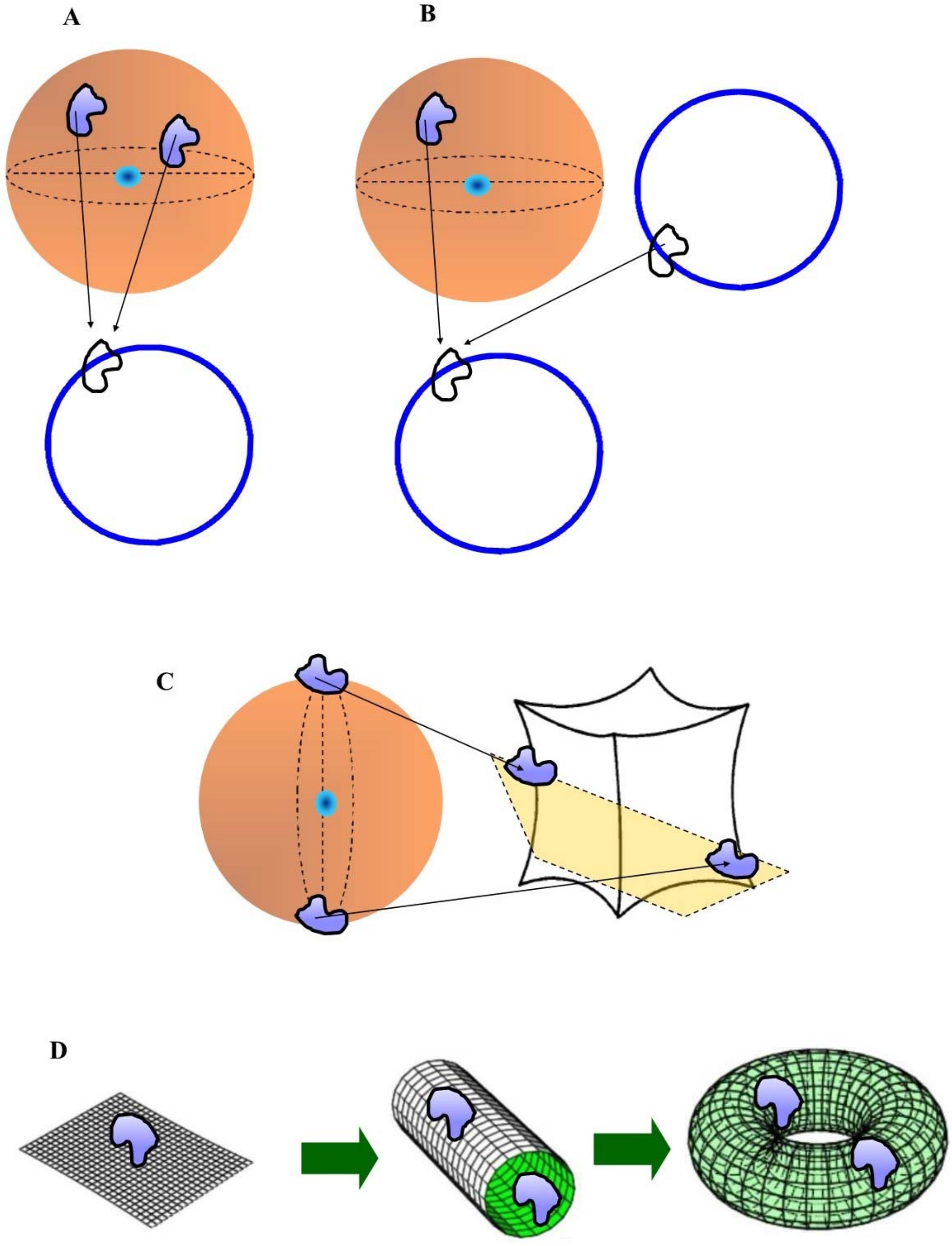

**Figure 4. Two features with signal matching do not need necessarily to be antipodal.** Indeed, the applications of BUT can be generalized also on non-antipodal points on the same sphere (**Figure 4A**), of non-antipodal points lying on two different structures (**Figure 4B**). **Figures 4C** and **4D** show how BUT applies for structures with different types of curvatures.

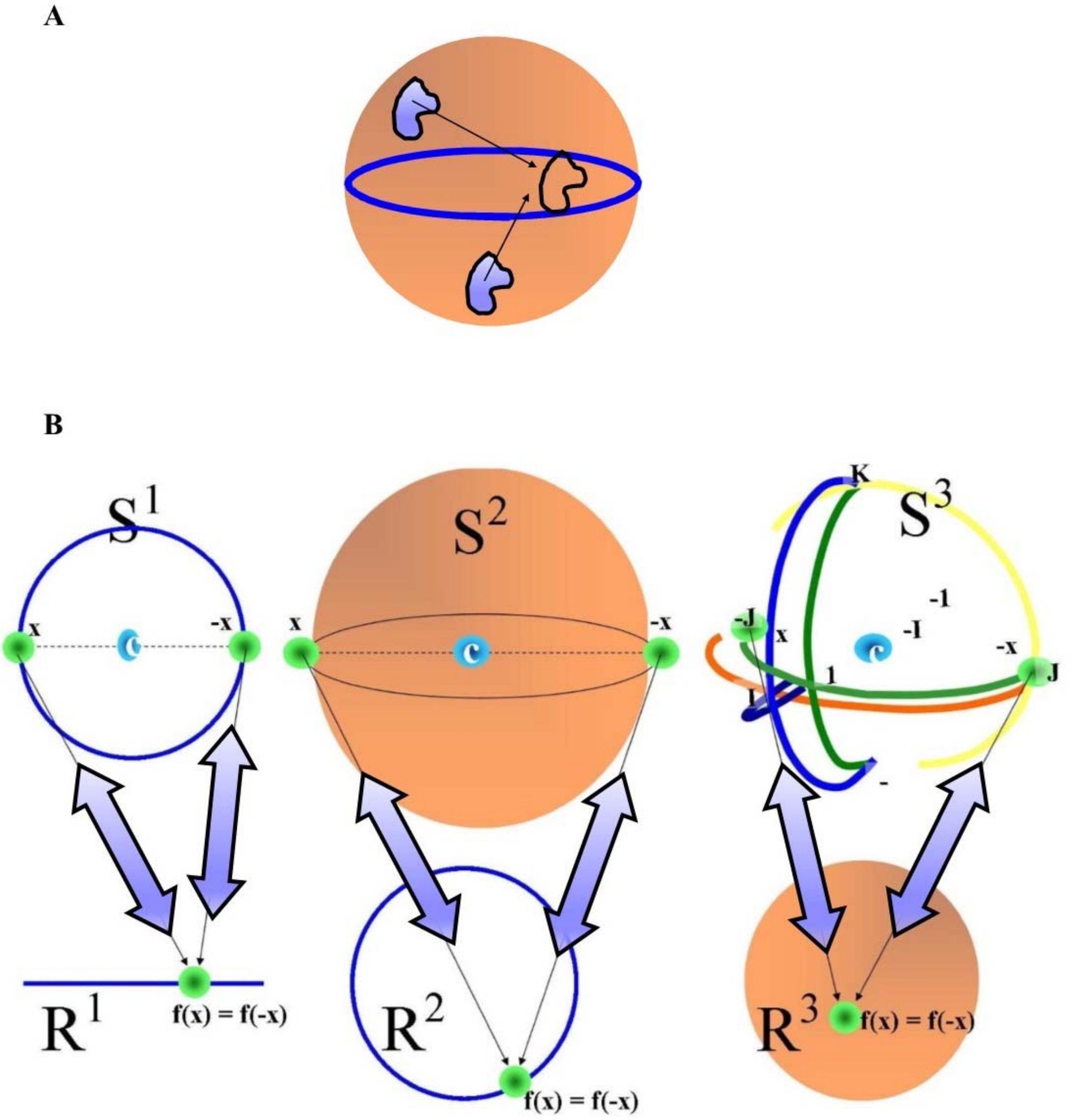

**Figure 5A.** The figure shows how a sphere may map on itself. **Figure 5B.** The way BUT acts on structures of different dimensions is displayed: a 2-D circle, a 3-D sphere and a 4-D hypersphere. A symmetry, e.g., two antipodal points in higher dimensions, is said to be "broken" when it maps to a single point in one dimension lower. Note that the mapping is reversible from higher to lower dimensions, and vice versa (double-sided arrows).

## 3. TOPODYNAMICS OF THE BRAIN-MIND OPERATIONAL ARCHITECTONICS

In this section, we will integrate the OA model with the mathematical framework of BUT and its variants. In particular, we will focus on the two levels of OA described in **Section 1**:

i. the low level of brain–mind operational architectonics, which deals with the formation of transient neuronal assemblies and their dynamics delimited by RTPs in relation to simple mind operations (subsections *3.1, 3.2*); and
ii. the high level of brain-mind operational architectonics, which deals with the formation of OMs through the synchronized operations of multiple transient neuronal assemblies in relation to complex mind operations, and their dynamics delimited by RTPs in relation to stream of consciousness (subsections *3.3*, *3.4*).

Along the discussion, we will also show how the real formalism for the description of BUT reinterpretations of the OA model can even reveal new insights not apparent from the neurophysiological standpoint by itself.

### *3.1. Nonlinear dynamics in brain-mind phenomenology*

The transient neuronal assemblies and metastable OMs defined in **Section 1** can be described in terms of abstract objects, and therefore assessed through the novel BUT variants described in **Section 2**. Indeed, BUT makes it possible to evaluate metastable brain dynamics in terms of projections from real to abstract phase spaces. Therefore, the nonlinear brain dynamics occurring during cognition and mentation can be assessed through more manageable linear techniques, in particular through projections and mappings. Indeed, BUT makes it possible for us to evaluate formally nonlinear brain dynamics, which occur during metastable cognition and consciousness, though in much simpler linear manner [71, 80]. In brief, nonlinear dynamics are frequently studied through logistic maps equipped with Hopf bifurcations, where intervals are dictated by Feigenbaum constants. Tozzi and Peters [71] introduced an approach that offers a linear explanation of nervous nonlinearity and Hopf bifurcations, in terms of algebraic topology. Hopf bifurcation transformations (the antipodal points) can be described as paths or trajectories on abstract spheres embedded in dimensions that stand for the Feigenbaum constant's irrational number [81, 82].

Isolated functions of local neuronal assemblies can be described in terms of abstract trajectories taking place on lower-dimensional structures, while globally coordinated activity among local neuronal assemblies stand for two (or more) trajectories with matching description taking place on higher-dimensional structures. Note that the term "dimension" does not stand here for a spatial one. In this case, we are assessing a "functional" dimension which describes the degree of complexity: the more dimensions, the more complexity and information. Therefore, going from lower to higher complexity dimensions, we achieve an increase in matching, superimposed cortical patterns. Such a dimensional topological approach makes it possible to quantify the attractive but elusive phenomenological account of the mind, in terms of cognitive neuroscientific operational and experimental terms. According to phenomenalists, every human individual originally accepts over her/him a) an ever-changing spatial environment composed of manifold parts dependent on one another; b) other human individuals making manifold describable statements and c) these statements are dependent upon the environment [83, 84]. When an individual becomes aware of some of the manifold parts of the environment outside or inside her/him, she/he states 'she/he is having an experience'. In a topological framework, the environment, the cortical layers and the describable statements can be embedded in different structures of various complexity dimensions, encompassing a different qualitative content and linked by projections. The object and its verbal counterpart stand for two antipodal points, while their matching description is achieved in the brain of the concrete human individual. It is noteworthy that the topological relationships among such three entities do not exist outside the triad: we cannot perceive the thoughts and the sensations of other people, but just project to them thoughts, statements and sensations analogous to ours. Subjective experiences relate to the environment just in the sense that the environment is able to modify the brain activity; thus, more accurately, they do not directly relate to the environment, but rather to complex brain activity variations.

In topological terms, the potential multi-variability of neuronal assemblies and their larger conglomerates is generated by the reciprocal mappings between lower-dimensional, short-term, local, metastable brain states and higher-dimensional, global ones. The topological concepts of mappings and projections helps us to assess metastability, if we take into account that lower-dimensional individual components of the brain/mind exhibit tendencies both to work autonomously, and to produce higher-dimensional, coordinated activity (equipped with matching descriptions). The concept of "projections" helps in elucidating a long-standing issue in science, namely, the cause-and-effect issue in the framework of OA. Current scientific experience is grounded on the tenets of causality, *e.g.*, the relation between two processes where the first, the cause, is assumed to be responsible for the second, the effect. It is believed that the brain infers the causes underlying sensory events [85], or, in other words, is able to understand the causes when just the effects are known. This inferencing is not limited to conscious and high-level cognition, but it is also performed continually and effortlessly in unconscious perception [86].

We argue that, if we evaluate different spatial and temporal neural phenomena in the light of the "sameness" and projections instead of causality, we achieve a novel view of brain events, in touch with OA model. Inference is replaced by affine connections and mappings: to make an example, perceived shapes map into others, such as a coffee cup maps to a torus. This is in touch with philosophers [87] and scientists [88-89] who believe that inference is just a trick of the mind, *i.e.*, the cause-effect relationships do not exist, being merely apparent correlations among events dictated by our natural, evolutionary instinct of mental association among conjoined events. In Hume's words, we tend to believe that things behave in a regular manner to the extent that behavior patterns of objects seem to persist into the future, and throughout the unobserved present. Chicharro and Ledberg [90] noticed that, because subsystems in the brain are often bidirectionally connected [91, 92], this means that interactions rarely should be quantified in terms of cause-and-effect.

### *3.2. Metastability and emergence of topodynamic properties*

The interaction among elementary, lower-dimensional neuronal systems, e.g., the neuronal assemblies, permits a synchronization of the stable microstates that takes place just in a higher dimension of complexity. This means that neuronal assemblies give rise to interactive information exchange of the essential variables, when projected to the higher-dimensional 'consensual decision' that is appropriate for each successive stage of behavioral/cognitive/mental performance (as specified in [26]). As we have already discussed in the **Section 1**, in the metastable regime of brain functioning, the interdimensional mapping between the two tendencies of autonomous fragmentation and cooperative integration among many distributed neuronal assemblies represents the coexistence of complementary, and not antagonistic, pairs [29, 30]. Therefore, a topological concept of metastability may refer to transient, non-stationary processes where complexity increases in higher dimensions, leading to an "enlargement" of shared information. Note that, contrary to the synergetic approach mentioned above, which "compresses" and "squeezes" the effective number of degrees of freedom to a few order parameters, our topological metastable approach "dilutes" and "scatters" the message, when going from lower to higher dimensions. Because increasing complexity accrues from antipodal points with matching description, trajectories embedded in lower dimensions display a decrease in the number of dimensions, and therefore of information, compared with the same trajectories embedded in higher dimensions [72]. In sum, in a topological fashion, repetitions of spatial–temporal brain patterns stand for trajectories that, although starting from very simple lower-dimensional nested components, nevertheless are able to capture higher-dimensional cortical dynamics as well as hierarchical complexities of brain activity.

OA and topological projection share important features: they are processes persisting over time, they are present in both brain and mind functioning, and they have a combinatorial nature. In **Section 1**, we stated that revealing the nonstationary structure of EEG field stands for the proper technique able to assess the dynamic changes of brain spatial–temporal patterns in a way that is isomorphic with cognitive and phenomenal levels [8-10]. The term *isomorphic*, in topological terms, does not refer to a one-to-one correspondence, but rather to a correspondence that is doubled in the higher level. The OA framework is centered around the notion of "operation" [4-6]. This apparently vague concept can be better assessed and quantified in topological terms, because operations become here simple projections and mappings among abstract manifolds. According to OA, the simplest mental/cognitive operations are presented in the brain in the form of lower-dimensional local fields produced by transient functional neuronal assemblies, while complex mental/cognitive operations are brought into existence by joining the simple operations in the form of the higher-dimensional OMs considering BUT principles [71].

Topology gives an unexpected substantiation to the concept of the OST level of brain organization [8-10] that is intervening between the IPST level where it literally resides, and the PST to which it is isomorphic. Indeed, the OA theory states that the OST level a) displays emergent properties independent from the IPST level's neuro-physiological and neuro-anatomical properties, and b) has a one-to-one correspondence with PST level that supervenes on, and ontologically inseparable from the OST level. This relationship is well elucidated in terms of BUT and its variants (**Figure 6**). As stated above, the only notable difference between OST and a topological framework is that the last does not display a one-to-one correspondence, but a one-to-two correspondence.

According to the OA theory, simple mental operations are equivalent to EEG quasi-stationary segments, the latter standing for the local fields generated by transient functional neuronal assemblies (**Figure 6**). Such quasi-stationary segments are marked by boundaries within each local EEG, in the form of sharp RTPs [4, 6]. Further, each RTP has a very short duration and can therefore be treated as a point or near-point, in guise of the points described by the classical BUT [20]. Note that BUT requires a function to be continuous, while the transition from one EEG segment to another reflects the moment of abrupt switching from one neuronal assembly's operation to another, and therefore is characterized by a transient loss of continuity [4, 6]. Such apparent discrepancy is easily explained (**Figure 6,** lower part). When, due to a change in environmental inputs, the continuity of a single perceived event is lost, a RTP occurs, and the phenomenal counterpart of this event vanishes in the brain/mind. In this brief instant, BUT does not hold anymore, and the inter-dimensional projections are temporarily lost. The continuity can be restored by a following incoming event, in order that novel, different matching projections are produced in the operational modules. Such a loss of continuity elucidates also the issue of the renewal process [41], which states that the temporary loss of projections resets the memory of the system.

### *3.3. Symmetry breaking, entropy, and information*

In terms of physical critical theory and symmetries, according to OA theory a biphasic process within the RTP between two consequent OMs takes place [10]. A brief episode, characterized by a drastic and abrupt increase in symmetries and degrees of freedom among the participating neuronal assemblies, is accompanied by a sudden increase in entropy, information and dimensionality.  This process is followed by quick episodes of symmetry breaks and reduction in the degrees of freedom of neuronal assemblies and rapid decrease in entropy, information and dimensionality [10]. The energetic variant of BUT comes into help in bringing more understand of these processes, because it states that, given a value of entropy in a lower dimension, it is twofold when projected in a higher dimension [25, 69, 74]. Godel's suggestion of abstract terms more and more converging to the infinity in the sphere of our understanding take us straight to a comparison with systems equipped with BUT antipodal points and regions. In terms of BUT, we achieve a progressive symmetry increase: starting from the symmetries endowed in the environment, we accomplish their substantial increase into the brain/mind [71].

The OA model is not anatomical, rather it is about the structure and dynamics of the electromagnetic field. Electromagnetic fields can be evaluated, through a topological approach, in the general terms of particle trajectories taking place on donut-like manifolds. Indeed, the OA model can be described in the guise of multi-dimensional tori, projecting and mapping among levels which display different spatio-temporal features. As stated above, in the evaluation of OA activities at every spatio-temporal scale, we are allowed to assess antipodal *regions* instead of antipodal *points* [17, 93]. BUT provides a way to evaluate changes of information among different anatomical and functional brain levels in a topological space, which is distinguished from purely functional or thermodynamical perspectives.

In the context of BUT we could name any brain location (with a particular neuronal assembly) a *worldsheet*, if all surrounding neuronal assemblies of its sub-regions contain at least one brain activity modelled as a string that describes a path followed by a moving particle. The term *cortical worldsheet* designates a region of the brain space completely covered by nervous activities [68] (**Figure 7**). A 2D plane worldsheet can be rolled up to form the lateral surface of a 3D cylinder, termed a *worldsheet cylinder* [93]. Further, a *cortical* worldsheet cylinder maps to a *cortical worldsheet torus*, formed by bending the former until the ends meet. In sum, such a flattened worldsheet maps to a worldsheet torus. This means that every brain activity can be assessed in terms of particle movements along the surface of multi-dimensional, donut-like toruses (**Figure 7**). In sum, BUT and its variants entail that every kind of high-dimensional brain activity predicted by OA can be described in terms of donut-like structures [71]. This methodological advance, already used in limited trials, could be useful in order to achieve a unvarying operationalization of the OA across inter- and intra-cellular levels [94].

### *3.4. Topological "duality" and OA levels*

As stated above, OA brain activity can be assessed either at functional micro-, meso- and macro-spatiotemporal scales of observation, or at intertwined levels with mutual interactions. We will show, based on topological findings, that nervous activities occurring in micro-levels project to single activities at macro-levels. This means that brain functions assessed at the higher scale of the whole brain necessarily display a counterpart in the lower ones, and vice versa. Consider Brouwer's fixed point theorem (FPT) [95]. Su [96] gives a nice illustration of the FPT: no matter how you continuously slosh the coffee around in a coffee cup, some point is always in the same position that it was before the sloshing began. And if you move this point out of its original position, you will eventually move some other point in the sloshing coffee back into its original position. In BUT terms, this means that not only we can always find a brain region containing an activity, but also that every activity comes together with another one, termed a *wired friend*. These observations lead to a *wired friend theorem*: every occurrence of a wired friend activity on the *n*+1 - dimensional structure maps to a fixed description, *e.g.*, to another activity that belongs to an *n*-dimensional structure (**Figure 7**) [68, 93]. The significance of this is that we can always find an OA activity, embedded in a higher-dimensional brain macro-level, which is the topological description of another activity, embedded in a lower dimensional brain micro-level; and vice versa. This leads to a novel scenario, where all the levels described by OA are *dual* under topological transformation. The term *dual* refers to a situation where two seemingly different physical systems turn out to be equivalent. If two techniques or phenomena are related by a duality, one can be transformed into the other, so that one phenomenon ends up looking just like the other one [97].

One of the most successful entropy-based theories of brain function, i.e., the free-energy principle, requires the brain activity taking place in an ergodic phase space [98]. In physics and thermodynamics, the ergodic hypothesis states that, over long periods, the time spent by a system in a region of the microstates' phase space with the same energy is proportional to the region's volume, so that all accessible microstates are equiprobable over a long period of time [99]. In other words, ergodicity is a random process characterized by the time average of one sequence of events being the same as the ensemble average [100, 101]. This also means that, in case of a Markov chain, as one increases the steps, there exists a positive probability measure at step *n* that is independent of the probability distribution at initial step *0* [102] However, widespread claims suggest that the brain is not fully ergodic. Indeed, many authors propose that the properties of brain fluctuations are inconsistent with the Markovian approssimation [103], that the mean-square distance travelled by brain particles displays anomalous diffusion [104] and that the brain is weakly non-ergodic, as some phase space region may take extremely long times to be visited. An OA/topological approach provides a potential solution of the problem of using ergodic information measures in the supposed non-ergodic "physical setting" of brain activity. Indeed, both ergodic and non-ergodic brain conditions can be "unified", e.g., treated in terms of algebraic topology, just changing the dimension of the structure under investigation. We provide an example: taking a 2D disk, and a particle that may "ergodically" travel along its whole surface, when one projects such 2D structure one dimension higher, a 3D sphere is achieved. The latter is equipped with two antipodal regions where the particle can travel. Therefore, in higher dimensions, the particles are constrained to move just in some zones of the structure, and ergodicity is partially lost. This means that, in the context of the OA/topological brain, the discrepancy between ergodic and non-ergodic spaces stands just for a "dual" theory.

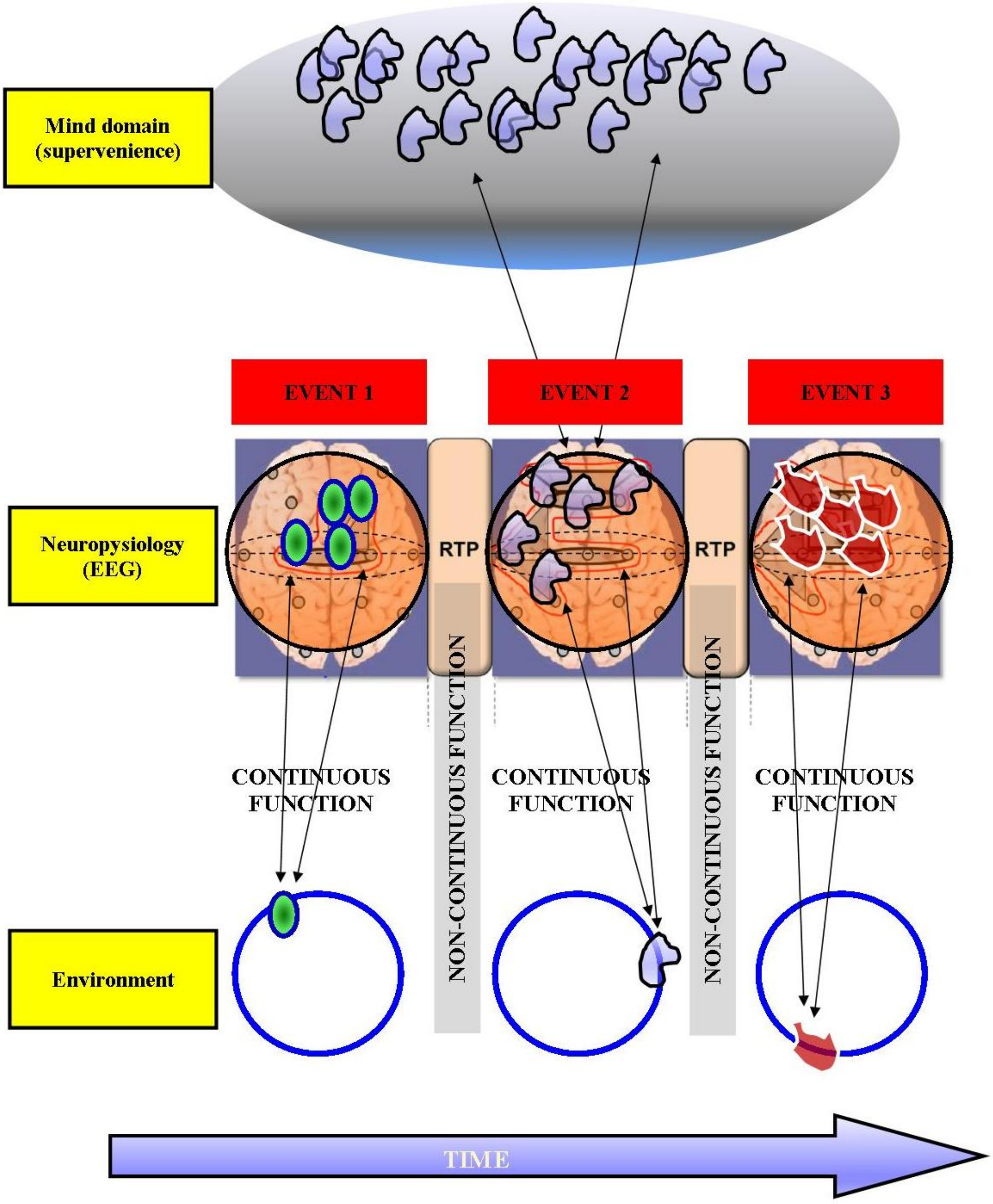

Figure 6. Topological description of operational modules. We accomplish a composite multi-dimensional system in which each local constituent exhibits close projective relationships with the others. **In the lower part of Figure**, different events (1, 2 and 3) occurring in the surrounding environment (in which the individual's brain/mind is embedded/embodied) stand for different functions (depicted by different shapes). Such shapes, when projected in the brain operational modules described by EEG traces, give rise to matching descriptions. Note that, despite BUT requires a continuous function, the continuity is lost, due to a change in the environmental input from event 1 to event 2, and then to event 3. In this case, a RTP occurs and the preceding event in a sequence vanishes. The continuity is then restored by the following event, and novel matching projections occur in the operational modules. **Middle part of the Figure**: if we just consider OA synchronization of neural assemblies in terms of matching descriptions that correlate (through pairwise entropy) spatially remote neurophysiological events, we achieve a novel description of operational modules. **The upper part of the Figure** displays a topological description of the higher levels of the brain–mind organization. Note that the displayed circles and spheres do not stand for different spatial dimensions, but for different abstract dimensions of increasing complexity.

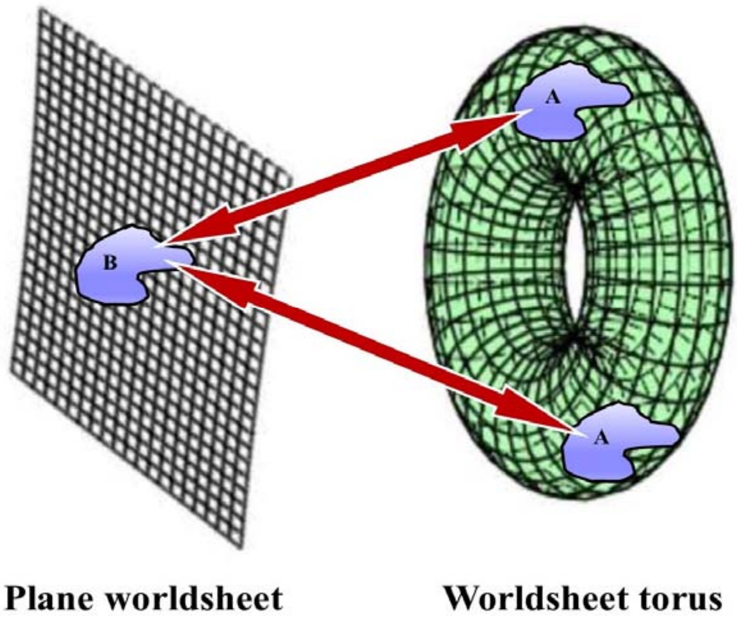

**Figure 7**. Samples of OA patterns, illustrated in guise of geometrical shapes, are represented by the regions A and B on different structures with diverse dimensions. Regions A and B are examples of antipodal activities which lie on structures with various dimensions.

## 4. PREDICTIONS AND NEW DIRECTIONS FOR FUTURE RESEARCH

The new topodynamic approach derived from interrelating OA and the BUT topological framework throws a new light on the evidences provided by functional studies of the brain, which indeed are a subject of intense and increasing research. In this section, we will mention the most relevant predictions and new research directions directly related to the theme of this review paper: the empirically observable dimensional/symmetry/entropy increases, the new topological operationalization of chaotic dynamics and synchronization in the brain, the unification of coarse-grained scales of observation, and – perhaps most importantly – a new topodynamic description of the cerebral production mechanisms of syntactics and semantics. Subsequently, a new path for novel computers with nodes built into higher dimensions is finally considered.

### *4.1. The observable dimensional/symmetry/entropy increase*

*The main prediction of the BUT formulation in relation to Operational Architectonics is that the brain displays, apart from the "classical" four spatio-temporal dimensions, also other functional dimensions, where the OA mechanisms might take place: an increase of functional dimensions occurs as a result of the* *environmental stimuli impinging upon the brain processes.* That is, the brain, rather than simply integrating sensory inputs and concentrating them into concepts as currently believed, appears to increase the complexity from the perceived physical object to the phenomenal object (“idea” of it) as suggested by the OA model [9, 72]. Indeed, phenomenal objects can be described as complex patterns of qualities which are spatially extended and bounded to each other to form a unified item (Gestalt) with a particular meaningful categorization (semantics) immediately present for the subject [10]. More specifically, the BUT model predicts that an increase in dimensions and symmetries occurs from the environment, through the separate neuronal assemblies in primary sensory areas, to the higher coalitions of such neuronal assembles in the form of Operational Modules (OMs), and that informational entropy in the primary sensory areas must be lower than in the higher associative ones.

This prediction was confirmed recently in a fMRI study where the entropy/information in zones of fMRI images was assessed, in order to evaluate whether, as predicted by our model, the entropy is higher in associative cortices than in the primary ones [72]. The results of that study suggest that the brain operational functioning lies in dimensions higher than the environment and that a message coming from external inputs does not concentrate in a local brain spot, but rather is presented by a nonlocal complex OMs, as dictated by the OA framework. In such an operational architecture, each of the complex OMs is not just a sum of simpler OMs or of many neuronal assemblies, but rather a natural union of abstractions about simpler OMs [9, 10]. Therefore, OMs have a rich combinatorial complexity and the ability to rapidly reconfigure themselves in accordance with the changing environmental input or mental activity.

Even though the OA is built on the electro-physiological field produced by the brain (EEG), some additional and complementary information could be extracted from the fMRI studies. And the BUT approach could be helpful here. A topological approach, which states that mappings among every brain signal and energetic information have the potential to be operationalized, makes it possible to extend the OA framework also in fMRI studies. As an example, let us consider the case of two brain states, one standing for a symmetry (the brain at rest), and another for a broken symmetry (the brain during a visual task). Our model sharply predicts the particular energetic values for both states (either in EEG or fMRI data), when they are functionally correlated [25, 74]. Indeed, according to BUT, a single micro-area with symmetry breaking (e.g., lower-dimensional neuronal assemblies), necessarily projects to TWO areas with preserved symmetry (*e.g.*, a high-dimensional OA module). The single area in lower dimensions and the two areas with matching description need to display the same values of entropy. This allows us to recognize which zones of the brain are correlated during symmetry breaks, *e.g.*, during the projective steps from higher to lower level of complexity, and vice versa.

Therefore, our framework is able to predict the following hypothetical results: if we find, during the visual task, say, three micro-areas with an entropy 1.08, we expect to find, during rest, six or more micro-areas with entropy=1.08. In sum, by knowing just the entropy values for each local EEG (or BOLD-activated area), we are allowed to correlate two different brain states, e.g., a lower-dimensional state with symmetry breaking and a higher-dimensional state with preserved symmetries. The phenomenon of spontaneous breakdown of symmetry and restoration of symmetry in the EEG signals has been widely documented and analyzed by Freeman and colleagues [105, 106]

### *4.2. The new topological operationalization of chaotic dynamics and synchronization in the brain*

*Another prediction is that the nonlinear dynamics required by OA can be tested through a topological approach. As stated above, nonlinear features of brain dynamics can be studied through logistic maps equipped with bifurcations, where intervals are dictated by Feigenbaum constants.* In such framework of brain functioning at the edge of chaos, BUT topodynamics comes into play. As demonstrated by Tozzi and Peters [71], embracing brain nonlinearity in the framework of the BUT means that brain activity bifurcation transformations (the antipodal points) can be described as paths or trajectories on abstract spheres equipped with Feigenbaum dimensions. Such an approach facilitates the evaluation of nonlinear brain activity dynamics through simpler linear techniques. Our framework provides a topological mechanism which explains the elusive brain phenomena at the edge of chaos predicted by OA, cast in a physical/formal fashion with the potential of being operationalized.

The OA theory proposes that the simultaneous occurrence of RTPs from different local EEG signals provides evidence of synchronization of simple operations performed by different neuronal assemblies in order to execute a particular complex operation [4, 10, 61, 107]. In topological terms, the synchronized quasi-stationary EEG segments separated by the RTPs are correlated with antipodal points in one dimension higher. The concepts of proximity (spatial as well as descriptive closeness, Peters [18]) and affine connections are also helpful in solving one of the problems raised by oscillations, e.g., how cooperation among so many "distant" sub-networks occurs seamlessly in real time. Thanks to the "continuous" mapping provided by BUT, a quantifiable explanation to the speed and balance of a system characterized by hierarchical and cross-hierarchical cooperating modules is achieved [45].

This explains the tendency of local EEG signals to be coordinated together within the same OM, in order to execute the macro-operation required to give rise to (a higher-dimensional level of) abstractness [9, 10]. In this sense, the concepts of parallel and serial processing may be just two sides of the same underlying mechanism, e.g., a synchrony of operations performed by mappings and projections. Therefore, we may conclude that each OM is a projective process, since it lasts as long as several operations are synchronized among each other.

### *4.3. Unification of coarse-grained scales of observation*

*The new framework permits a topological duality among the different levels of OA, because it holds for all the types of spatiotemporal brain activities, independent of their inter- and intra-level relationships, strength, magnitude and boundaries. The generalization achieved allows the assessment of every possible OA's brain level, independent on its scale, specific features and local boundaries.* Our topological investigation reveals that there must be at least one element in common among brain spatiotemporal activities that lie in different functional dimensions. In other words, there exists an assessable and quantifiable correspondence between micro-, meso- and macro-levels of OA brain levels. OA brain levels will always have some element in common: they do not exist in isolation, rather they are part of a large interconnected whole, with which they interact.

Therefore, the distinction among different coarse-grained levels of nervous activity does not count anymore, because nervous function at small, medium and large scales of neural observation turn out to be topologically equivalent. It allows a useful simplification in the assessment of brain activity. Because projections between dimensions describe neural phenomena spanning from the smallest to the highest scales, the distinction among different coarse-grained scales does not count anymore: indeed, nervous activity is topologically the same at small, medium and large scales of observation. In sum, two activities with matching description, embedded in two brain locations of different levels, display the same features. Activities with matching ends (regions) in different cortical areas might also help to throw a bridge, for example, between sensation and perception. In the same guise, the relationships between the spontaneous and the evoked activity of the brain [108] take now a new significance: they become just two sides of the same coin, made of topologically-bounded dynamics.

### *4.4. Cerebral production mechanisms of syntactics and semantics*

*Another important prediction and new direction for future research involves the mechanisms of syntactics and semantics in the brain. According to the OA-topological model, it might be argued that the human brain processes syntactic and semantic elements through a change in functional/spatial dimensions.* When presented with a proposition, the brain straightforwardly understands its grammar and discriminates whether it is correct or wrong, true or false. Unlike computers, the brain is able to identify signs of sequences in terms of both syntactic symbols and semantic meaning. It could be demonstrated, based on the current literature, that a testable algebraic topological approach gives helpful insights into the brain's computational activity during semantic recognition. Indeed, recent suggestions allow us to hypothesize that the semantic properties of a proposition, corresponding to the higher levels described by OA, are processed in brain dimensions higher than the syntactic ones. Further, we might show how, in a fully reversible process, the syntactic elements embedded in Broca's area can project into scattered semantic cortical zones. Thereafter, the presence of higher functional dimensions gives rise to an increase in the informational content of propositions. In order to explain that dimensional increase, we account for the dictates of the novel versions of the BUT and FPT.

In a very simplified view, the syntactic processing that occurs at one level of brain activity is projected to a higher level. This gives rise to semantic recognition, which encompasses more information. In our novel framework, syntax displays fewer dimensions than semantics in the central nervous system, so that semantic inputs have increased complexity. Therefore, the first task is to determine if the brain contains different dimensions, while the second is to describe nervous dynamics as vectors or tensors in pluri-dimensional phase spaces. The term ‘brain *dimension*’ may reflect either: a) levels of functional activity, b) or the number of components that pinpoint anatomical connections between cortical areas. The functional approach, based on complex network analysis of brain signals, assesses the multi-dimensionality of neural space in a brain model viewed as a complex dynamical system [109-111]. Apart from the *canonical* three dimensions, such technique is also capable of assessing other neural features such as frequency and magnitude, with each representing other possible dimensions [109].

The anatomical approach to brain dimensionality evaluates cortical locations [71, 80]; and as recent advances indicate, brain trajectories may display four spatial dimensions – instead of the canonical three – during cortical spontaneous activity [79, 80]. Such trajectories are described in terms of torus-shaped objects. Animals are shown to navigate by reading out a simple population vector of grid cell activity across multiple spatial scales [94]: this means that the behavior of population vectors, each lying in different anatomical dimensions embedded on instances of the functional torus, predicts neural and behavioral correlation to grid cell readout. An algorithm and model was developed to study how complex networks are organized by higher-order connectivity patterns, so as to reveal hubs and geographical elements not readily achieved by other methods. For further details, see [112]. It was demonstrated that information propagation units exhibit rich higher-order organizational structures.

In the same vein, real neural networks are not just random combinations of single network layers, but are instead organized in specific ways; these are dictated by hidden geometric correlations between layers, which permit the detection of multidimensional communities [113]. This is crucial for the BUT new models, because such multi-dimensionality enables trans-layer link prediction so that connections in one layer can be predicted by observing the hidden geometric structures of another layer. Defining “strings” as paths followed by a particle moving through space, our BUT model [74] provides a new formal framework for the distinctive characterization of semantics and syntactics. The brain localizations of syntactic and semantic processing as described in literature seem to agree with our topological model. The left anterior language area, e.g. Broca’s area (left Brodmann area 44), is crucial for syntactic processing in speech production and perception [114, 115]. This is despite the wide variability between individuals and the presence of diverse circuits not dedicated to just a single type of linguistic information processing [116]. While syntax seems to be localized in relatively fewer and smaller brain areas, semantics is instead scattered throughout vast areas of the cortical surface [117]. This would agree with a model encompassing BUT variants and FPT that regards semantic concepts as multiple antipodal strings on a torus surface, and syntactic concepts as single strings located in specific brain zones. This also means that syntactic elements lie on higher brain dimensions than syntactic ones. According to the dictates of our BUT model, single strings capture less information than their matching descriptions embedded in higher projections. This is a seminal concept, and it is not necessarily intuitive. Note also that semantic information is not aggregated, but instead it is scattered in the brain. Syntactic symbols represent single sets of objects, while semantic meanings represent numerous sets of objects with matching description.

*To sum up, an OA/topological or topodynamical approach makes it possible to build a feasible explanation for semantics processing in the brain. But it also suggests a new path for novel computers with nodes built into higher dimensions. Indeed, the fact that brain nodes can be embedded in higher dimensional spaces suggests the possibility of manufacturing computers equipped with pluri-dimensional nodes endowed in 3D geometric spaces. Yet a computer can be hypothesized having connections that are topologically 4D, although embedded in a simple 3D space. Such computers, implemented in ordinary 3D space, but equipped with nodes with the same number of neighbors as points in a 4D cube, could have the capability to perform semantic operations, due to more degrees of freedom.*

## CONCLUSIONS

In conclusion, we have achieved a topodynamic description of the main elements of the operational architectonics of brain-mind functioning. BUT and its extensions provide a methodological approach which makes it possible for researchers to study metastable brain/mind in terms of projections from real to abstract phase spaces, which is crucially important for building accurate computational and mathematical models of the brain-mind functioning (see also [118, 119]). The real, measurable activity of the brain can now be described in terms of paths occurring on abstract structures. This leads to a consideration of affinities among multiple signals of brain activity, characterized as antipodal points on multi-dimensional structures endowed in abstract spaces. A shift in conceptualizations is evident in the BUT approach: the onset of the neuronal assemblies and their interactions within the operational modules is described in terms of sameness, closeness instantiated by their feature value vectors (e.g., amplitude, duration, and so on). The invaluable opportunity to treat elusive mental activities in terms of topological structures makes it possible for us to describe experience in the language of powerful analytical structures. Embracing OA model systems in the framework of algebraic topology [119, 120] means that transformations (the antipodal points or regions) can be described as paths or trajectories on “abstract” structures: the BUT perspective makes possible a system’s property located in the real space (the physical milieu’s geometric space) to be translated to an abstract space (called the topological configuration space) and vice-versa, enabling us to achieve maps from one level to another. This puts in a new light the system interactions described by OA theory [4-10], in terms of affine connections and proximity (either spatial or descriptive closeness) among signals, in order to explain, for example, how network communities integrate or segregate information during complex mental activity. In sum, topodynamics of the metastable brain supplies us with richly sufficient statistics that helps us to elucidate the mechanisms of human brain/mind functioning. The paths described by BUT and its variants elucidate how the tight coupling among the different neural activities described by the OA model gives rise to self-organization where spatiotemporal patterns in one part of the brain, which are in charge of receiving and interpreting signals from other cortical parts, are closely intertwined at every spatio-temporal level. The AO model assessed in the light of a topological framework, therefore achieving the new topodynamic perspective, becomes one of the central information processing strategies of the advanced nervous system, including its semantic and syntactic ‘computational’ processes. The phenomenon seems traceable from an evolutionary point of view as well: whether the evolution of complexity in central nervous systems recapitulates essential topodynamic developments becomes an exciting possibility that could also open new research directions.

## REFERENCES

1) Lehar S. 2003. Gestalt isomorphism and the primacy of subjective conscious experience: A Gestalt bubble model. Behavioral and Brain Sciences 26:357–408.
2) Khrennikov A.Yu. 2004. Information dynamics in cognitive, psychological and anomalous phenomena. Ser. fundamental theories of physics. Dordrecht: Kluwer.
3) Perlovsky LI. 2006. Modeling field theory for higher cognitive functions. Hershey, PA: Idea Group.
4) Fingelkurts AA, Fingelkurts AA. 2001. Operational architectonics of the human brain biopotential field: Towards solving the mind-brain problem. Brain and Mind 2:261–296.
5) Fingelkurts AA, Fingelkurts AA. 2005. Mapping of the brain operational architectonics. In: Chen FJ (ed) Focus on brain mapping research. Nova Science Publishers Inc, New York, pp 59–98.
6) Fingelkurts AA, Fingelkurts AA. 2008. Brain-mind operational architectonics imaging: technical and methodological aspects. Open Neuroimag J 2:73–93.
7) Fingelkurts AA, Fingelkurts AA. 2015. Operational Architectonics Methodology for EEG Analysis: Theory and Results. Neuromethods 91:1–59.
8) Fingelkurts AA, Fingelkurts AA, Neves CFH. 2009. Phenomenological architecture of a mind and operational architectonics of the brain: the unified metastable continuum. New Math Nat Comput 5:221–244.
9) Fingelkurts AA, Fingelkurts AA, Neves CFH. 2010. Natural world physical, brain operational, and mind phenomenal space-time. Phys Life Rev 7(2):195–249.
10) Fingelkurts AA, Fingelkurts AA, Neves CFH. 2013. Consciousness as a phenomenon in the operational architectonics of brain organization: Criticality and self-organization considerations. Chaos Solitons Fractals 55:13–31.
11) Bressler SL, Kelso JAS. 2001. Cortical coordination dynamics and cognition. Trends Cogn Sci 5(1):26–36.
12) Uhlhaas PJ, Singer W. 2012. Neuronal dynamics and neuropsychiatric disorders: toward a translational paradigm for dysfunctional large-scale networks. Neuron 75:963–980.
13) Tognoli E, Kelso JA. 2014. Enlarging the scope: grasping brain complexity. Front Syst Neurosci 8, 122.
14) Willard S. 1970. General topology, Dover Pub., Inc., Mineola, NY. xii +369pp, ISBN: 0-486-43479-6 54-02, MR0264581.
15) Krantz SG. 2009. A guide to topology, The Mathematical Association of America, Washington, D.C.
16) Manetti M. 2015. Topology, Springer, Heidelberg. DOI 10.1007/978-3-319-16958-3.

17) Naimpally, SA, Peters, JF. 2013. Topology with Applications. Topological Spaces Near and Far. Foreword by I.A. Taimanov. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2013. xvi+277 pp. ISBN: 978-981-4407-65-6, MR 3075111.
18) Peters JF. 2016. Computational Proximity. Excursions in the Topology of Digital Images. Intelligent Systems Reference Library 102. Switzerland: Springer Int. Pub. DOI: 10.1007/978-3-319-30262-1.
19) Peters JF, Tozzi A. 2016. Quantum Entanglement on a Hypersphere. Int J Theoret Phys, 1–8. doi:10.1007/s10773-016-2998-7.
20) Borsuk M. 1933. Dreisätzeüber die n-dimensionaleuklidischesphäre, FundamentaMathematicae XX: 177–190.
21) Marsaglia G. 1972. Choosing a Point from the Surface of a Sphere. Annals of Mathematical Statistics 43 (2): 645–646.
22) Matoušek J. 2003. Using the Borsuk–Ulam Theorem. Lectures on Topological Methods in Combinatorics and Geometry. Berlin Heidelberg: Springer-Verlag.
23) Beyer WA, Zardecki A. 2004. The early history of the ham sandwich theorem. American Mathematical Monthly 1:58-61.
24) Moura E, Henderson DG. 1996. Experiencing geometry: on plane and sphere. Prentice Hall. ISBN 978-0-13-373770-7.
25) Tozzi A, Peters JF. 2017. A Symmetric Approach Elucidates Multisensory Information Integration. Information 8(1), 4; doi:10.3390/info8010004
26) Fingelkurts AA, Fingelkurts AA. 2004. Making complexity simpler: Multivariability and metastability in the brain. Int J Neurosci 114:843–862.
27) Kelso, J. A. S. 1991. Behavioral and neural pattern generation: The concept of Neurobehavioral Dynamical System (NBDS). In H. P. Koepchen (Ed.), Cardiorespiratory and Motor Coordination. Berlin: Springer-Verlag.
28) Kelso JAS. 1995. Dynamics patterns: the self-organization of brain and behaviour. MIT Press, Cambridge, MA.
29) Kelso JAS. 2009. Coordination dynamics. In: Meyers RA (ed) Encyclopedia of complexity and systems sciences. Springer, Berlin, pp 1537–1564.
30) Kelso JAS, Engstrøm D. 2006. The complementary nature. MIT Press, Cambridge.
31) Kelso JAS, Tognoli E. 2007. Toward a complementary neuroscience: Metastable coordination dynamics of the brain. In:
32) Haken H. 1983. Synergetics: An introduction. 3rd ed. Berlin: Springer-Verlag.
33) Haken H. 1999. What can synergetics contribute to the understanding of brain functioning? In C. Uhl (Ed.), Analysis of neurophysiological brain functioning (pp. 7–40). Berlin: Springer-Verlag.
34) Freeman WJ. 2004. Origin, structure, and role of background EEG activity. Part 1. Analytic amplitude. Clin Neurophysiol 115:2077–2088.
35) Miwakeichi F, Martinez-Montes E, Valdes-Sosa PA, Nishiyama N, Mizuhara H, Yamaguchia Y. 2004. Decomposing EEG data into space–time–frequency components using Parallel Factor Analysis. NeuroImage 22:1035–1045.
36) Buzsaki G, Anastassiou CA, Koch C. 2012. The origin of extracellular fields and currents—EEG, ECoG, LFP and spikes. Nat Rev Neurosci 13:407–420.
37) Bak P. 1996. How nature works: the science of self-organized criticality. New York: Springer.
38) Jensen HJ. 1998. Self-organized criticality: emergent complex behavior in physical and biological systems. Cambridge: Cambridge University Press.
39) Bianco G, Grigolini P, Paradisi P. 2005. Fluorescence intermittency in blinking quantum dots: renewal of modulation? J ChemPhys 123:174704.
40) Silvestri L, Fronzoni L, Grigolini P, Allegrini P. 2009. Event- driven power-law relaxation in weak turbulence. Phys Rev Lett 102:014502.
41) Allegrini P, Menicucci D, Bedini R, Fronzoni L, Gemignani A, Grigolini P, et al. 2009. Spontaneous brain activity as a source of ideal 1/f noise. Phys Rev E 80:061914.
42) Bak P, Tang C, Wiesenfeld K. 1987. Self-organized criticality: an explanation of the 1/f noise. Phys Rev Lett 59:381–384.
43) Bak P. 1997. How nature works. Oxford, UK: Oxford University Press.
44) Novikov E, Novikov A, Shannahoff-Khalsa D, Schwartz B, Wright J. 1997. Scale-similar activity in the brain. Phys Rev E 56:R2387–9.
45) Linkenkaer-Hansen K, Nikouline VV, Palva JM, Ilmoniemi RJ. 2001. Long-range temporal correlations and scaling behavior in human brain oscillations. J Neurosci 21(4):1370-1377.
46) Freeman WJ. 2003. Evidence from human scalp electroencephalograms of global chaotic itinerancy. Chaos 13(3):1067–1077.
47) Plenz D, Thiagarajan TC. 2007. The organizing principles of neuronal avalanches: cell assemblies in the cortex? Trends Neurosci 30:101–110.
48) Gong P, van Leeuwen C. 2009. Distributed dynamical computation in neural circuits with propagating coherent activity patterns. PLoS Comput Biol 5(12):e1000611.

49) Werner G. 2009. Viewing brain processes as critical state transitions across levels of organization: neural events in cognition and consciousness, and general principles. BioSystems 96:114–119.
50) Allegrini P, Paradisi P, Menicucci D, Gemignani A. 2010. Fractal complexity in spontaneous EEG metastable-state transitions: new vistas on integrated neural dynamics. Front Physiol 1:128.
51) Paradisi P, Allegrini P, Gemignani A, Laurino M, Menicucci D, Piarulli A. 2012. Scaling and intermittency of brain events as a manifestation of consciousness. AIP Conf Proc 1510:151–161.
52) Fingelkurts AA, Fingelkurts AA. 2006. Timing in cognition and EEG brain dynamics: discreteness versus continuity. Cogn Process 7:135–162.
53) Churchland PS, Sejnowski T. 1992. The computational brain. MIT, Cambridge.
54) Fodor JA, Pylyshyn ZW. 1988. Connectionism and cognitive architecture: a critical analysis. Cognition 28:3–71.
55) James W. 1890. The principles of psychology, Vol. I. New York: Dover.
56) Fingelkurts AA, Fingelkurts AA, Ivashko RM, Kaplan AYa. 1998. EEG analysis of operational synchrony between human brain cortical areas during memory task performance. Vestn Moskovsk Univ Ser 16 Biol 1:3–11. in Russian.
57) Fingelkurts AA, Fingelkurts AA, Krause CM, Kaplan AY, Borisov SV, Sams M. 2003. Structural (operational) synchrony of EEG alpha activity during an auditory memory task. NeuroImage 20(1):529–542.
58) Fingelkurts AA, Fingelkurts AA, Krause CM, Möttönen R, Sams M. 2003. Cortical operational synchrony during audio–visual speech integration. Brain Lang 85:297–312.
59) Fingelkurts AA, Fingelkurts AA, Kivisaari R, Pekkonen E, Ilmoniemi RJ, Kähkönen SA. 2004. Local and remote functional connectivity of neocortex under the inhibition influence. NeuroImage 22(3):1390–1406.
60) Fingelkurts AA, Fingelkurts AA, Kivisaari R, Autti T, Borisov S, Puuskari V, et al. 2007. Opioid withdrawal results in an increased local and remote functional connectivity at EEG alpha and beta frequency bands. Neurosci Res 58(1):40–49.
61) Fingelkurts AA, Fingelkurts AA, Kivisaari R, Autti T, Borisov S, Puuskari V, et al. 2009. Methadone may restore local and remote EEG functional connectivity in opioid-dependent patients. Int J Neurosci 119(9):1469–1493.
62) Prigogine I, Stengers I. 1984. Order out of chaos: man’s new dialogue with nature. New York: Basic Books.
63) Stanley HE. 1987. Introduction to phase transitions and critical phenomena. Oxford, UK: Oxford University Press.
64) Marro J, Dickman R. 1999. Nonequilibrium phase transitions in lattice models. Cambridge, UK: Cambridge University Press.
65) Haken H. 2000. Information and self-organization: a macroscopic approach to complex systems. New York: Springer.
66) Borsuk M. 1958-1959. Concerning the classification of topological spaces from the standpoint of the theory of retracts. Fundamenta Mathematicae XLVI:177–190.
67) Borsuk M. 1969. Fundamental retracts and extensions of fundamental sequences, Fundamenta Mathematicae 1:55–85.
68) Peters JF, Tozzi A. 2016. String-based Borsuk-Ulam theorem and wired friend theorem. arXiv preprint 1605, no. 02987v3.
69) Peters JF, Tozzi A. 2017. From abstract topology to real thermodynamic brain activity. Cognitive Neurodynamics, DOI: 10.1007/s11571-0-17-9431-7. in press.
70) Weeks JR. 2002. The shape of space, IInd edition. New York-Basel : Marcel Dekker, inc.
71) Tozzi A, Peters JF. 2016b. A topological approach unveils system invariances and broken symmetries in the brain. J Neurosci Res, 94(5), 351-365. DOI: 10.1002/jnr.23720.
72) Peters JF, Tozzi A, Ramanna S, Inan E. 2017. The human brain from above: an increase in complexity from environmental stimuli to abstractions. Cognitive Neurodynamics, DOI: 10.1007/s11571-0-17-9428-2, in press.
73) Mitroi-Symeonidis FC. 2015. Convexity and Sandwich Theorems. European Journal of Research in Applied Sciences 1 (March): 9–11.
74) Tozzi A, Peters JF. 2016c. Symmetries, Information and Monster Groups before and after the Big Bang. Information, 7(4), 73; doi:10.3390/info7040073.
75) Weyl H. 1982. Symmetry. Princeton: Princeton University Press. ISBN 0-691-02374-3.
76) Weyl H. 1918. Raum—Zeit—Materie. Vorlesungen über allgemeine Relativitätstheorie (Space—Time—Matter). Reprinted in H. Weyl (1952). Space—Time—Matter, trans. By H.L. Brose, Dover Pubs, Inc.
77) Roldán É, Martínez I, Parrondo JMR, Petrov D. 2014. Universal features in the energetics of symmetry breaking. Nature Physics, 10(6), 457–461. http://doi.org/10.1038/nphys2940.
78) Peters JF, Tozzi A, Ramanna S. 2016. Brain Tissue Tessellation Shows Absence of Canonical Microcircuits. Neuroscience Letters 626: 99–105. doi:10.1016/j.neulet.2016.03.052.
79) Peters JF, Ramanna S, Tozzi A, Inan E. 2017. Frontiers Hum Neurosci. BOLD-independent computational entropy assesses functional donut-like structures in brain fMRI image. doi: 10.3389/fnhum.2017.00038.
80) Tozzi A., Peters JF. 2016a. Towards a fourth spatial dimension of brain activity. Cogn Neurodyn, 10 (3), 189-99 PMID: 27275375. DOI: 10.1007/s11571-016-9379-z.
81) Kim I-S. Extensions Of The Borsuk.Ulam Theorem. J. Korean Math. Soc. 34 (1997), No. 3, pp. 599–222
82) Schleicher D. 2007. Hausdorff dimension, its properties, and its surprises, The American Mathematical Monthly 114, no. 6, 509–528, MR2321254.

83) Avenarius R. 1908. Kritik Der Reinen Erfahrung von Richard Avenarius. Lepzig: C.R. Reisland.
84) Russo Krauss C. 2015. L'empiriocriticismo di Richard Avenarius tra psicofisiologia e teoria della consocenza, in R. Avenarius, Il concetto umano di mondo, Morcelliana, Brescia.
85) Parise CV, Spence C, Ernst MO. 2011. When correlation implies causation in multisensory integration. CurrBiol 22(1):46-49.
86) Körding KP, Beierholm U, Ma WJ, Quartz S, Tenenbaum JB, Shams L. 2007. Causal inference in multisensory perception. PLoS One 26;2(9):e943.
87) Hume D. 1739–40. A Treatise of Human Nature: Being an Attempt to introduce the experimental Method of Reasoning into Moral Subjects. Penguin Classics; New Ed edition (31 Oct. 1985)
88) Libet B, Gleason CA, Wright EW, Pearl DK. 1983. Time of conscious intention to act in relation to onset of cerebral activity (readiness-potential). The unconscious initiation of a freely voluntary act. Brain 106 (Pt 3):623-642.
89) Bengson JJ, Kelley TA, Zhang X, Wang J-L, Mangun GR. 2014. Spontaneous Neural Fluctuations Predict Decisions to Attend. J CognNeurosci. 26(11):2578-2584.
90) Chicharro D, Ledberg A. 2012. When two become one: the limits of causality analysis of brain dynamics. PLoS One 7(3):e32466. doi: 10.1371/journal.pone.0032466.
91) Friston K. 2008. Hierarchical models in the brain. PLoS computational biology 4, e1000211
92) Barrett LF, Simmons WK. 2015. Interoceptive predictions in the brain. Nat Rev Neurosci 16(7):419-429.
93) Peters JF. 2016. Two forms of proximal physical geometry. Axioms, sewing regions together, classes of regions, duality, and parallel fibre bundles. Advances in Math.: Scientific J. 5(2), 241-268, arXiv: 1608.06208v4.
94) Marijuán PC, del Moral R, Navarro J. 2013. On eukaryotic intelligence: signaling system's guidance in the evolution of multicellular organization. Biosystems, 114(1):8-24. doi: 10.1016/j.biosystems.2013.06.005. Epub 2013 Jul 12.
95) Volovikov, Yu A. 2008. Borsuk-Ulam Implies Brouwer: A Direct Construction Revisited. American Mathematical Monthly 115 (6): 553–56. doi:10.2307/2975293.
96) Su FE. 1997.Borsuk-Ulam implies Brouwer: A direct construction. Amer. Math. Monthly 104 (9): 855–859, MR1479992.
97) Zwiebach, Barton (2009). A First Course in String Theory. Cambridge University Press. ISBN 978-0-521-88032-9
98) Friston K. 2010. The free-energy principle: a unified brain theory? Nat Rev Neurosci 11(2):127-138.
99) Barth A. Vorlesungen über Gastheorie. Leipzig: 1898. OCLC 01712811. ('Ergoden' on p.89 in the 1923 reprint.)
100) Walters P. 1982. An Introduction to Ergodic Theory, Springer, ISBN 0-387-95152-0
101) Alaoglu, L., & Birkhoff, G. (1940). General ergodic theorems. The Annals of Mathematics, 41(2), 293-309.
102) Feller W. 1971 An introduction to probability theory and its applications, vol. 2, Wiley
103) Fraiman D, Chialvo DR. 2012. What kind of noise is brain noise: anomalous scaling behavior of the resting brain activityfluctuations. Frontiers in Physiology, 3:307.
104) Bianco S, Ignaccolo M, Rider MS, Ross MJ, Winsor P, Grigolini P. Brain, music, and non-Poisson renewal processes.Phys. Rev. E 75, 061911 – Published 21 June 2007.
105) Freeman WJ, Vitiello G. 2006. Nonlinear brain dynamics as macroscopic manifestation of underlying many-body field dynamics. Phys Life Rev 3:93-118.
106) Freeman WJ, Vitiello G. 2010. Vortices in brain waves. Int J Mod Phys B 24:3269-3295.
107) Fingelkurts AA, Fingelkurts AA. 2010. Alpha rhythm operational architectonics in the continuum of normal and pathological brain states: current state of research. Int J Psychophysiol 76:93–106.
108) Tozzi A, Zare M, Benasich AA. 2016. New Perspectives on Spontaneous Brain Activity: Dynamic Networks and Energy Matter. Frontiers in Human Neuroscience. doi:10.3389/fnhum.2016.00247.
109) Kida T, Tanaka E, Kakigi R. 2016. Multi-Dimensional Dynamics of Human Electromagnetic Brain Activity. Front Hum Neurosci. 19;9:713. doi: 10.3389/fnhum.2015.00713. eCollection 2015.
110) Giusti C, Ghrist R, Bassett DS. 2016. Two’s company, three (or more) is a simplex. Algebraic-topological tools for understanding higher-order structure in neural data. Journal of Computational Neuroscience, 41; 1:1–14.
111) Simas T, Chavez M, Rodriguez PR, Diaz-Guilera A. 2015 An algebraic topological method for multimodal brain networks comparisons. Front Psychol. 6;6:904. doi: 10.3389/fpsyg.2015.00904. eCollection 2015.
112) Tozzi A, Peters JF. 2017. Towards Topological Mechanisms Underlying Experience Acquisition and Transmission in the Human Brain. J.F. Integr. psych. behav. doi:10.1007/s12124-017-9380-z.
113) Kleineberg K-K, Boguñá M, Serrano MA, Papadopoulos F. 2016. Hidden geometric correlations in real multiplex networks. Nature Physics, in press. doi:10.1038/nphys3812.
114) Skeide MA, Friederici AD. 2016. The ontogeny of the cortical language network. Nature Reviews Neuroscience 17, 323–332. doi:10.1038/nrn.2016.23
115) Stromswold K., Caplan D, Alpert N, Rauch S. 1996. Localization of syntactic comprehension by positron emission tomography. Brain and Language 52, 452–473.
116) Sahin NT, Pinker S, Cash SS, Schomer D, Halgren E. 2009. Sequential processing of lexical, grammatical, and phonological information within Broca’s area. Science 326, 5951. 445-449.
117) Huth AG, de Heer WA, Griffiths TL, Theunissen FE, Gallant JL. 2016. Natural speech reveals the semantic maps that tile human cerebral cortex. Nature. 28;532(7600):453-8. doi: 10.1038/nature17637.

118) Sengupta B, Tozzi A, Cooray GK, Douglas PK, Friston KJ. 2016. Towards a Neuronal Gauge Theory. PLoS Biol 14(3): e1002400.
119) Kozma R, Perlovsky L (eds). 2007. Neurodynamics of higher-level cognition and consciousness. Springer, Heidelberg.
120) Krantz SG. 2010. Essentials of Topology with Applications. CRC Press, Boca Raton, FL, MR2554895.