*Below is the unedited draft of the article that has been accepted for publication*
*(© Progress in Biophysics and Molecular Biology, 2018, V. 140, P. 133-141.*
*DOI: https://doi.org/10.1016/j.pbiomolbio.2018.05.010)*

# Syntax Meets Semantics During Brain Logical Computations

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**Abstract**

The discrepancy between syntax and semantics is a painstaking issue that hinders a better comprehension of the underlying neuronal processes in the human brain. In order to tackle the issue, we at first describe a striking correlation between Wittgenstein's Tractatus, that assesses the syntactic relationships between language and world, and Perlovsky's joint language-cognitive computational model, that assesses the semantic relationships between emotions and "knowledge instinct". Once we have established a correlation between a purely logical approach to the language and computable psychological activities, we aim to find the neural correlates of syntax and semantics in the human brain. Starting from topological arguments, we suggest that the semantic properties of a proposition are processed at a level that is higher dimensional than the functional level used by the brain for syntactical properties. In a fully reversible process, the syntactic elements embedded in Broca's area project into multiple scattered semantic cortical zones. The presence of higher functional dimensions gives rise to the increase in informational content that takes place in semantic expressions. Therefore, diverse features of human language and cognitive world can be assessed in terms of both the logic armor described by the Tractatus, and the neurocomputational techniques at hand. One of our motivations is to build a neuro-computational framework able to provide a feasible explanation for brain's semantic processing, in preparation for novel computers with nodes built into higher dimensions.

**Keywords**: Borsuk-Ulam; Brouwer; computation; meaning; truth; syntactic

## INTRODUCTION: TROUBLES WITH SEMANTICS

S*yntax* assesses the relationships among the elements of linguistic expressions (Carnie, 2006): a property of the proposition is syntactic, provided it depends only on the symbols. Syntax is akin to grammar, which also includes phonology and orthography, such as the sounds and spellings of words (Martinich, 1996). Also, *syntax* refers to the presentation of symbols in a sequence that is valid as based on correctness of designated true/false values (Béziau, 2010). In turn, s*emantics* assesses the relationships between expressions and extra-linguistic truths (Nerbonne, 1996). A property of the assertion is semantic, provided it depends only on the meaning.

Semantics stands also for the logical "information flow" of linguistic expressions, such as in a sequence of arguments (Cruse, 2004). The problem is that syntactic properties are verifiable by computerized algorithms, while semantic properties are not, because the latter are based on the non-physicalist notion of truth. The issue of the unrelatability of syntax and semantics was first raised by Ludwig Wittgenstein. Indeed, the triumph of syntax depicted his seminal Tractatus Logico-Philosophicus (Wittgenstein, 1921) turned out to be incapable to solve the difficult logical and philosophical problems of syntactic versus semantics. That's why the so called "second" Wittgenstein (1953) rejected the philosophical scenario depicted in his own book and introduced the (almost) opposite semantic theory of "linguistic jokes". The issue was further complicated by the incompleteness theorems (Gödel, 1931) that demonstrated how, if one is limited to syntactic approaches, she is not capable to obtain truths. This means that, given a group of arithmetical axioms, there is always a true arithmetic proposition that cannot be demonstrated starting from the axioms, if we admit just syntactic methods (Segre, 1994; Mancosu, 1998). Hence, a paradox occurs: syntactic methods are inadequate to understand the complete properties of the same model that is easily understandable through semantics. This leads us to neuroscience and neurocomputation: despite the human thought is effortlessly able to grasp the semantic notion of truth, such non-physicalist truth cannot be verified by computers. Therefore, a) if unquestioned and consistent syntactic methods are used, then all truths cannot be proved; b) if the uncertain semantic models endowed in our brain are used, then all arithmetic truths possibly could be known, but the correctness of such models is ambiguous.

The challenge here is: could the pure logical and syntactic content described by the Tractatus be useful in the current neuroscientific debate? How do logical limitations look like, contemplated through the lenses of the neurocomputational models at hand? Is it feasible to achieve a computational implementation of the syntax, in order to make the Tractatus' tenets mathematically assessable and describable in semantic terms? Could computational syntax and brain semantics be treated through mathematical tools able to remove combinatorial complexity?

Herein we show how a semantic theory that links language and cognition, i.e., Perlovsky's joint language-cognitive model (Perlovsky and Sakai, 2014; Perlovsky, 2016), coupled with the topodynamic tenets of the recently proposed brain operational architectonics (Tozzi et al., 2017), is able to describe the very basic syntactic assumptions of the Tractatus. Further, we propose a new topological-computational approach so as to bridge between syntactic and semantic expressions in the human brain. An added motivation is to advance towards the construction of semantics-solving machines.

## A LOGIC LINK BETWEEN SYNTACTIC AND SEMANTIC ISSUES

Here we show how two apparently unsuited approaches (the one equipped with syntactic and logical features, the other with semantic and computational structures) can be fruitfully joined, in order to logically describe the correlations between the language and the world.

**“Deep” syntactics: Wittgenstein’s logical correlations between the language and the world**.

Wittgenstein establishes in his *Tractatus* an isomorphism, a correlation and a quantitative articulation between the world (and its basic components) and language (and its basic components) (**Figure 1**). These relationships are located in a fixed and immutable grid, the logical armor, embedded in a logical space where isomorphisms between the world and the language occur, through one-to-one-into projections. Starting from the simplest terms of this relationship, we have, on the world’s side, the things, *i.e*., the objects, simple and immutable. They stand for the unbridgeable border of the analysis and make up the irreducible substance of the world. From the language side, such simplest terms of the relationship stand for names, *i.e*., words equipped with a meaning.  From the world’s side, the second level stands for the states of things, i.e., atomic facts, state of affairs, logical connections of things and links of objects. An objects’ configuration is various, changing, irregular. Then, from the language side, the states of things stand for elementary propositions, *i.e*., the atomic representations of the objects: they are independent one from each other, equipped with sense and characterized by truthful meaning. From the world side, the third higher step are the facts, i.e., configuration relationships, portrayals, logic of reality, subsistence of the states of things. From the language side, they stand for the composite propositions, *i.e*., the propositions termed “molecular” that are the logic products of elementary propositions. They display the logic properties of language, but do not say anything else. Indeed, they have neither meaning, nor content; they just cope with the logical shape, unveiling the word armor. They are true and describe the real, despite the nettlesome fact that they do not define a primary, naïve, direct view. Their sense depends on the elementary propositions, because they are truth functions of the latter. The highest step stands, by one side, for the world itself: it is a limited whole, the logic space of the facts, a pure formal universality, a unique, essential structure that fixes up what happens. Note that the totality prevails on the simple level. The world’s counterpart, at the highest level, is the language itself, which stands for the limit of the world. Life, the subject, the I, is located outside the world and language. Where the subject is located, nothing occurs. The subject, being out of the world, stands for the border, the non-content of the world. From the subject’s location, the occurring experiences are available not through the ordinary syntax, rather through the immediacy of feeling. Accordingly, the role of philosophy is to proceed from the top to the bottom, from language to names. It is a top-down approach that starts from the higher level of the world and allows the analysis of the lower-levels, i.e., the states of things.

During the following years, Wittgenstein realized that his model was causing him troubles.The elementary, “atomic” structures of the Tractatus stand for both the names and the corresponding things in the world.  Yet, how could one be sure that names and things are the very basic structures of the language and the world, respectively? How could one be sure that names and things cannot be split in further elementary components?  How can I say that this is a “sword”, and not just a collection of pieces, such as the blade, and so on? Therefore, the “second” Wittgenstein concluded that the human brain copes with indescribable “semantic” issues of words and things.  Indeed, when presented with a proposition, the brain straightforwardly understands its grammar and discriminates whether it is correct or wrong, true or false.  Unlike computers, the brain is able to identify signs of sequences in terms of both syntactic symbols and semantic meaning. How do cope with the discrepancy between the logical syntax and the semantics endowed in our brains? Here, Perlovsky's account comes into play.

**"Deep" semantics: Perlovsky's physical correlations between language, cognition and emotions**.

Perlovsky has developed a multi-level model of neural architecture that permits a computational description for the mental mechanisms of language, cognition and emotions (Perlovsky, 2011). A hetero-hierarchical structure occurs from sensory signals at the bottom, to highest concepts' representations at the top (Friston, 2010; Tozzi et al., 2017), while learning and recognition proceed in parallel with object perception. Concept representations stand for mental models of objects and situations. During visual perception, for instance, a mental model of the object stored in memory projects an image (top-down signals) onto the visual cortex, where it matches an image projected from retina (bottom-up signals). The brain, according to Perlovsky, makes use of "dynamic logic", i.e., a fuzzy process "from vague-to-crisp" (Ilin et al., 2014), where the degree of fuzziness is automatically set, corresponding to the accuracy of the learned models. In the process of learning, models become more and more accurate and association variables become crisper and crisper, so to increase and maximize the similarity between top-down and bottom-up messages. At each hierarchical level, concept-models encapsulate the mind's knowledge, generating top-down neural signals that interact with inputs (i.e., bottom-up signals). Knowledge is not static, rather a continuous process of adaptation and learning that takes place in order to understand the ever-changing surrounding world. Humans are equipped with what Perlovsky calls the "knowledge instinct", an inborn need, a drive to fit top-down and bottom-up signals that continuously improve our cognition. Given that cognition is inextricably linked with human language (Perlovsky, 2013), every mental semantic representation consists of two model aspects: cognitive and linguistic. Meanings are created by symbol-processes in the mind. Language plays a special role because, through its ability to accumulate cultural knowledge and communication among people, it provides grounding for abstract model-concepts at the higher levels of the mind hierarchy. Therefore, semantic issues, such as cognitive activities, language faculties, emotions, and also motivations, are mathematically assessable through a series of well-established equations, the "joint language-cognitive models" (**Figure 1**). These equations maximize similarity without combinatorial complexity, due to the matching vagueness of similarity measures and the uncertainty of the model. In the process of learning and understanding input signals, models are adapted for their better representation, so that the similarity between models and signals increases. This increase in similarity satisfies the knowledge instinct and is mentally perceived as aesthetic emotions (Schoeller and Perlovsky, 2016).

Opposite to Wittgenstein's first model, Perlovsky's account might be considered a fully semantic one. Indeed, he proposes Dynamic logic (DL), i.e., a variant to overcome limitations of "classical" logic (the one used by Wittgenstein, after Frege's and Russell's modifications). Classical logic is static (e.g., "this is a chair"), while DL is a process "from vague-to-crisp", or in other words, from a vague semantic representation, model, decision and plan, to crisp ones. DL could be viewed as a dynamic form of fuzzy logic that automatically sets a degree of fuzziness corresponding to the extent of the acuity each learned model with acuity measured on a continuous scale from 0 to 1. For technical readers, the mathematical account of Perlovsky's procedure is provided in Perlovsky (2011).

**When Wittgenstein meets Perlovsky.**

As stated above, in the later period of his life, the so called "second" Wittgenstein dismissed the tenets of his own Tractatus. He realized that his syntactic "atomic facts" cannot be accurately described, because they just depend of the (semantic) communication context in which they are formulated (therefore, he started to talk about "linguistic jokes"). Here we will show that the late Wittgenstein was wrong, because the syntactic treasures of the Tractatus still hold true, when implemented through the Perlovsky's original account of semantics through fuzzy logic (Yoder, 2009). Perlovsky's tools are appropriate not just for the evaluation of every layer of mental hierarchies, cognitive as well as language models, but also for the Tractatus' content. Indeed, the first Wittgenstein's logical account of syntax and the Perlovsky's computational treatment of semantics display an unnoticed relationship. As illustrated in **Figure 1**, a clear superimposition occurs between their accounts of language and cognition. In touch with the Tractatus, the joint language-cognitive model is organized in parallel hierarchies of language (words, texts) and cognitive models (world's mental representations). Near the bottom of these hierarchies, words refer to objects. Higher up, complex texts refer to complex situations. This means that words within texts refer to objects within situations (because situations are collections of objects), so that reference at higher levels corresponds to the words-objects relationships at lower levels. The highest levels (the most general models) can be predicted starting from the lower ones, because symbols are psychical processes that create meanings: they involve conscious as well as unconscious activities, concepts, emotions and models learned from culture, language and cognition. Therefore, Perlovsky's concept-model, built in order to maximize correspondence between the algorithm internal structure (semantic knowledge) and object recognition, establishes that the whole first Wittgenstein's' account is mathematically assessable and implementable through computational techniques at hand.

In sum, the tenets of the two influential frameworks may be superimposed, so that strict correspondences can be drawn between their single components and levels. This means that the two completely different approaches describe the same dynamics, and that syntax and semantics are not as far as we might believe. We may, therefore, draw the conclusion that syntax and semantics display combining features.

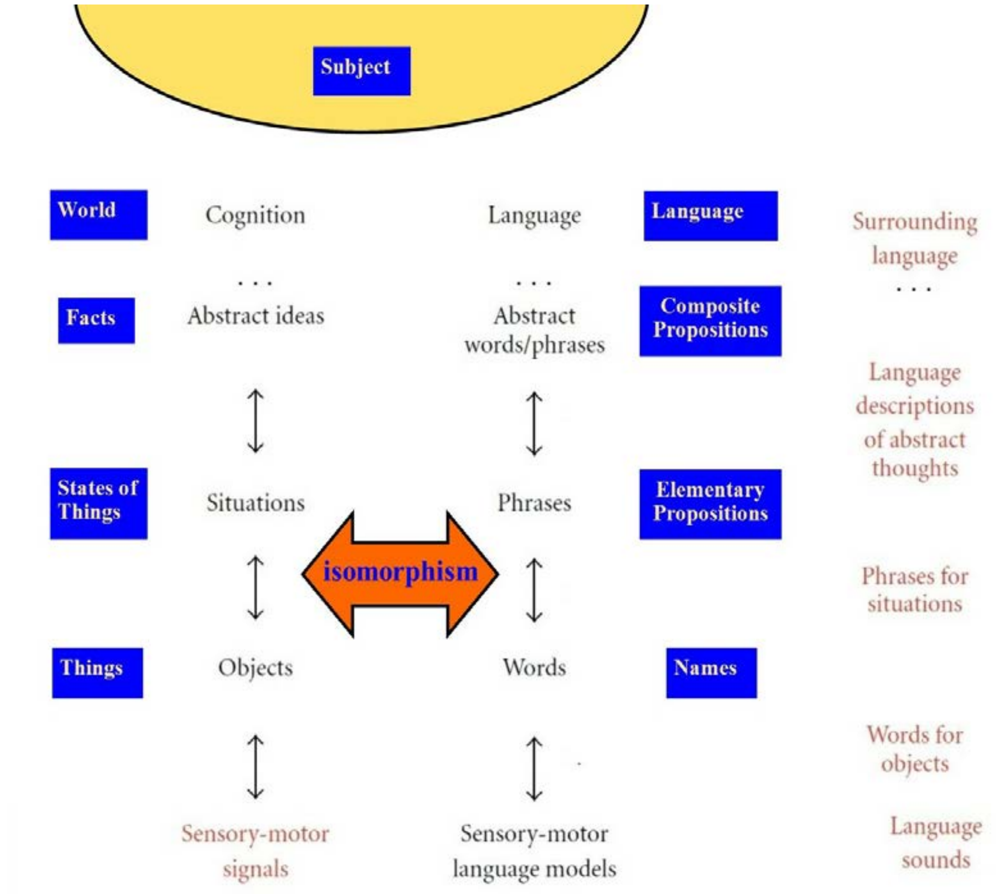


**Figure 1.** Wittgenstein's isomorphic model (in dark squares) superimposed to Perlovsky's joint language-cognitive model. The latter depicts an entire hierarchy from sounds to words and phrases, until the highest concepts existing in culture. Both the models display parallel hierarchies of language and cognition (or world), where lower-level concepts and things project to higher levels equipped with meanings relevant to situations.

## LOOKING FOR THE NEURAL CORRELATES OF SYNTAX AND SEMANTICS

In the previous section, we described a rather unexpected correlation between Wittgenstein's syntax and Perlovsky's semantics. In other words, we established a relationship between a purely logical approach to the language and testable psychological issues, such as emotions and subjectivity. The next step is to look for the neural correlates that would allow the unification of syntactic and semantic operations in the human brain. We need to transfer the above-mentioned syntactic and semantic matching account to the realm of the (natural and/or artificial) networks assessed by neuroscience. As we will illustrate in the next paragraphs, the possibility of such amalgamation lies in algebraic topology.

### Entering topological issues.

Our central hypothesis is that the human brain, due to its unique operational architectonics (Fingelkurts and Fingelkurts, 2001, Fingelkurts et al., 2010), processes syntactic and semantic elements through a change in functional and spatial dimensions. This hypothesis is rooted in the work of Walter Freeman related to intentionality. Throughout his studies, Freeman provided a comprehensive analysis of brain intentionality. He used the intentional action-perception cycle (Freeman, 1997; Freeman, 2012; Freeman and Vitiello 2006), in which individual neurons, neuron populations, and their supporting glia and vasculature architectures generate the cortical neurodynamics responsible for perception, cognition, and behavior (Kozma and Freeman, 2009). According to this approach, brains are not passive receivers of information, but rather are the active agents that search for sensory input (Freeman, 2008; Capolupo et al., 2013). In topological terms, we may state that Wittgenstein's syntactic processing occurs at one operational level of brain activity, but projection is mapped to a higher functional/operational level where it is further processed (Tozzi et al., 2017). This extra processing gives rise to Perlovsky's semantic recognition, which encompasses far more information than the original Wittgenstein model. An explanation may be framed via new topological developments based on the Borsuk-Ulam and the Brouwer fixed-point theorems. Accordingly, we will hypothesize that the syntactic elements of a language lie on an *n-manifold*, while the semantic elements of a language lie on an *n+1 manifold*. The Borsuk-Ulam Theorem (BUT) states that there is a continuous function that maps antipodal points with matching descriptions into an n-dimensional space of real numbers. For technical readers, see: Borsuk (1933), Matousek (2003), Crabb and Jaworowski (2013), Tozzi and Peters (2016a), Peters (2016). Continuous projections from an *n*-dimensional sphere to a *n*-dimensional Euclidean space lead to a string-based instance of BUT, termed strBUT. We consider a geometric structure with the characteristics of a *string* (Peters and Tozzi, 1016). By definition, a string on the surface of an n-sphere is a line that represents the path traced by a particle moving along its surface. In an abstract geometric space, a string, also termed *worldline* (Olive, 1987; Olive and Landsberg, 1989), is a region of space with either bounded or unbounded lengths. In evaluating a string-based BUT, we take into account antipodal sets instead of antipodal points (Petty, 1971). Indeed, in a point-free geometry (Di Concilio, 2013; Di Concilio et al., 2017), regions replace points as the primitives. If we assess a worldline in terms of a spatial region on the surface of an *n*-sphere, or in an *n*-dimensional normed linear space, strings can be defined as antipodal (**Figure 2A**). Strings are *antipodal*, provided the regions encompassing the strings belong to disjoint parallel hyperplanes which have no points in common. A region is called a *worldsheet* if every one of its sub-regions contains at least one string. The term *worldsheet* designates a nonempty region of a space completely covered by strings, in which every member is a string. A 2D plane worldsheet can be rolled up to form the lateral surface of a 3D cylinder, named a worldsheet cylinder. Further, a worldsheet cylinder maps to a worldsheet torus, formed by bending the former until the ends meet (**Figure 2A**). Hence, a flattened worldsheet maps to a worldsheet cylinder, and a flattened worldsheet cylinder maps to a worldsheet torus. This means that a bounded worldsheet cylinder is homotopically equivalent to a worldsheet torus.

The strings on different worldsheets are antipodal and descriptively near, i.e., they share a matching description. There is however a difference between the strings embedded in worldsheets of diverse dimensions. The higher the dimension of the worldsheet, the more information is encompassed in the strings on the same region, because of the higher number of coordinates. Strings contain more information than their projections in lower dimensions. This means that strBUT permits the evaluation of system features in higher dimensions—this in turn increases the amount of detectable information. For example, the 3D shape of a cat can be implied by the contour of its 2D shadow. For the process in reverse with complex points, a decrease in the number of dimensions implies that the lower dimensional space is occupied by simpler points. Therefore, strBUT provides a method to evaluate changes of information in a topological rather than in a thermodynamic framework: a projection mapping to a dimension at a lower level contains multiple mappings to a dimension at a higher level.

Next, we consider Brouwer's fixed point theorem (FPT). This theorem states that every continuous function from an *n*-sphere into an *n*+1 sphere has at least one fixed point (Brouwer, 1906). For example, FPT applies to a disk-shaped area to guarantee the existence of a fixed point behaving as a sort of whirlpool attracting moving particles. A coffee cup illustration of the FPT is given by Su (1997). Regardless of how the coffee is stirred without spilling, one point is always in the same position as occupied before the stirring commenced. Even more, if that point is moved out of its original position, then some other point is displaced back into its original position. In terms of the strBUT, the FPT means that an n-sphere always contains a string. Each string has a particular shape and also comes along together with another string, called a *wired friend* (Peters, 2016). This leads to the *wired friend theorem*: the instance of a wired friend string with a particular shape on the structure $S^n$ maps to a fixed description as another string that belongs to another structure $S^{n+1}$ (Peters and Tozzi, 2016). The wired friend is recognizable by its shape, because the shape of a string is the silhouette of a wired friend string. We obtain maps of wired friend strings, projecting, from the 2D worksheet, to the cylinder worldsheet, to the torus worldsheet. Therefore, we can always find a structure of a higher dimension containing a string that is a description of another string mapped from a structure of lower dimension, and vice versa. In the next section, we show how this theorem has important consequences in the study of syntactic and semantic brain processing.

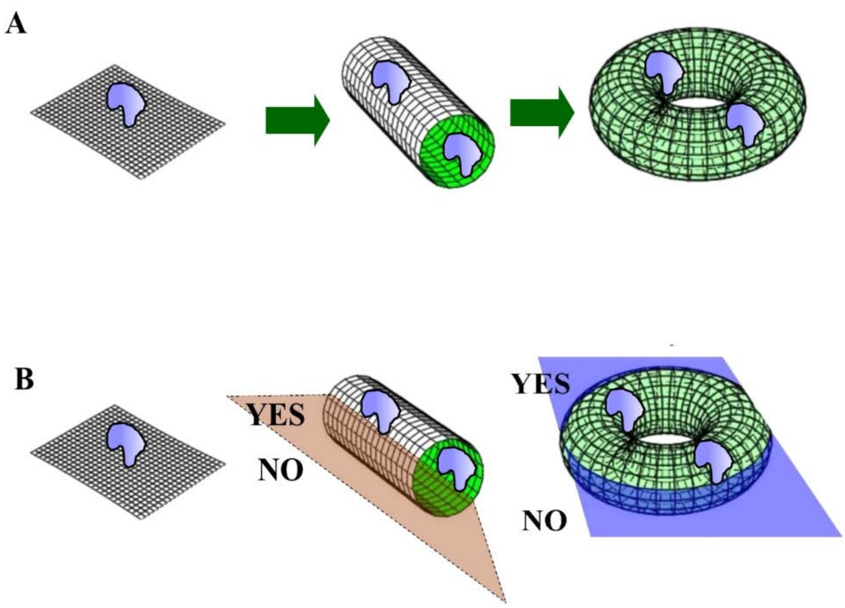


**Figure 2A**. **String mappings according to StrBUT**. A string (blue spot) maps to a higher dimension, from the 2D worldsheet to the 3D worldsheets of the cylinder or torus. The strings on the three different structures are equipped with matching descriptions and are thus antipodal. Note that, for the classical BUT, a single string on the 2D worksheet maps to two opposite strings on the 3D worldsheets.

**Figure 2B**. **Syntactic and semantic counterparts of strBUT**. On the left, the blue spot stands for a bidimensional syntactic construct. Moving to one higher dimension achieves a third dimension. This further dimension stands for the semantic concept of a truth value as YES or NO to mean proof or contradiction. Thus, strings gain novel information when projected into structures of a higher dimension.

**Why topology provides the logical chasm between syntactics and semantics**.

By means of strBUT and FTP, we can approach language in terms of both syntactics and semantics. We hypothesize that the syntactic elements of a language lie on an *n*-manifold, while the semantic elements of a language lie on an *n*+1 manifold. A syntactic proposition becomes a semantic argument when brain mechanisms project the proposition of one dimension onto a higher dimension for the argument, and the reverse process also applies. In logical terms, we define the syntactic proposition as $(p = q)^n$, where n = 2, and the semantic argument as $(p=q)^{n+1}$. At the higher dimension, we introduce the truth function such that the semantic argument returns True or False. For example, consider the syntactic preposition "the-skin-is-tender" and the semantic argument equipped with a truth function of True (or Yes): "*the skin is tender*". The truth value, of course, could be False (or Not) in the case of a disease affecting the skin. We write $(\text{the-skin-is-tender})^n$ and $(\textit{the skin is tender})^{n+1}$, where *n* stands for the abstract dimensions in which the proposition is embedded. The two propositions also stand for two groups, where the *n*+1 group encompasses the *n*-group as a subgroup. The *n*+1 group as equipped with a truth function (Yes or No) cannot be detected fully, if we look at just the *n*-group. This is because the *n*-group exhibits lesser dimensions than the *n*+1 group. When going from a semantic argument to a syntactic proposition, information may be lost at the proposition level due to the lower number of coordinates as mapped. This means that the semantic group is assessed first in the higher dimension than in the syntactic one. In topological terms, syntactic operations take place when strings as linguistic prepositions are placed in a 2D worldsheet, while semantic operations take place when strings as linguistic arguments are placed in 3D worldsheets for cylinders and toruses. Therefore, a group equipped with the further functionality of True or False is formed through a projective, continuous mechanism (**Figure 2B**). In terms of the classical BUT, semantic elements encompass not just one, but two antipodal strings, equipped with a matching description. Such framework is useful when we assess the cortical counterparts of propositions. We contrast the two points lying on a higher dimension and the single point lying on a lower dimension. The main difference is that the two antipodal points as lying on a higher dimension form a group of themselves. That means that semantic processing can be assessed and operationalized solely if we take into account a context equipped with dimensions higher than that of syntactic symbols. Furthermore, when the dimension *n* increases, syntactic structures are compared with models existing one dimension higher to show similarities. The exponent *n*+1 simply means a higher spatial or functional dimension in the brain. As we will discuss later, computers equipped with nodes dedicated for higher spatial dimensions might be able to process semantic arguments too.

**Why topology provides the neural chasm between syntactics and semantics**.

In the above Sections, we have hypothesized that the human brain processes syntactic and semantic elements through a change in functional or spatial dimensions. The syntactic processing that occurs at one level of brain activity is projected to a higher level; this gives rise to semantic recognition, which encompasses more information. Here we point at possible physical brain counterparts. In the central nervous system, syntax displays fewer dimensions than semantics, so that semantic inputs have increased complexity. The first task is to determine whether the brain contains different dimensions or not. The term brain *dimension* may reflect either a) functional/operational activity, or b) anatomical/functional connections between cortical areas in the form of spatio-temporal patterns. The functional approach a), based on complex network analysis of brain signals, assesses the multi-dimensionality of neural space in a brain model viewed as a complex dynamical system (Kida et al, 2016; Giusti et al., 2016; Simas et al., 2015). This approach describes nervous dynamics as vectors or tensors in pluri-dimensional phase spaces (Stemmler et al., 2015; Benson et al., 2016; Kleinberg et al., 2016). Apart from the *canonical* three dimensions, is also capable of assessing other neural features, such as frequency and magnitude, each one representing other possible dimensions (Kida et al, 2016). The anatomical approach b) to brain dimensionality evaluates cortical locations (Tozzi and Peters 2016a, 2016b); and as recent advances indicate, brain trajectories may display four spatial dimensions during cortical spontaneous activity, instead of the canonical three. Such trajectories are described in terms of torus-shaped objects. The brain representation of syntactic and semantic processing as described in literature seems to confirm our topological model. The left anterior language areas, e.g. Broca's area (left Brodmann area 44), are crucial for syntactic processing in speech production and perception (Skeide and Friederici, 2016; Stromswold et al., 1996). This is despite the wide variability between individuals and the presence of diverse circuits not dedicated just to a single type of linguistic information processing (Sahin et al., 2009). While syntax seems to be localized in relatively fewer and smaller brain areas, semantics is instead scattered throughout vast areas of the cortical surface (Huth et al, 2016). This is in agreement with our approach, that brings to existence a semantic hierarchy of linguistic thought, through the increasing complexity of joint operations that link many functional and transient neuronal assemblies in the brain (Benedetti et al., 2010), and a model encompassing StrBUT and the fixed-point theorem that regards semantic concepts as multiple antipodal strings on a torus surface—and syntactic concepts appearing as single strings located in specific brain zones (**Figure 3**). This also means that semantics lies in a brain with higher dimensions than syntax.

According to the dictates of our strBUT model, single strings capture less information than their matching descriptions embedded in higher projections. Note that semantic information is not aggregated, rather is scattered and hierarchically organized in the brain, as brain operational architectonics predicts (Benedetti et al., 2010; Fingelkurts et al., 2003, 2010). Every part of this hierarchical, nested operational architecture is simultaneously present to every other part/component, forming a spatial co-presence of semantics (Fingelkurts et al., 2012). Syntactic symbols represent single sets of objects, while semantic meanings represent numerous sets of objects with matching description. By extension, we predict that, during the human brain development, the semantic processes precede the syntactic ones. Indeed, children exhibit a slow developmental segregation of syntax from semantics (Skeide and Friederici, 2016). The human embryo can already distinguish vowels *in utero*, but grammatical complexity is usually not fully mastered until at least 7 years (Piaget and Inhelder, 1958;

Pinker, 2007). In the first three years, children rapidly acquire bottom-up processing skills, primarily implemented in the temporal areas. In a further stage until adolescence, top-down processes emerge gradually with the increasing structural connectivity of the left inferior frontal cortex (Skeide and Friederici, 2016). This means that children are equipped with more semantics than adults. Nevertheless, these valuable insights provided by our topological tools need a further development, in order to incorporate the extra cognitive-emotional complexities of real or "deep" semantics; and it is here that Perlovsky's model comes into play.

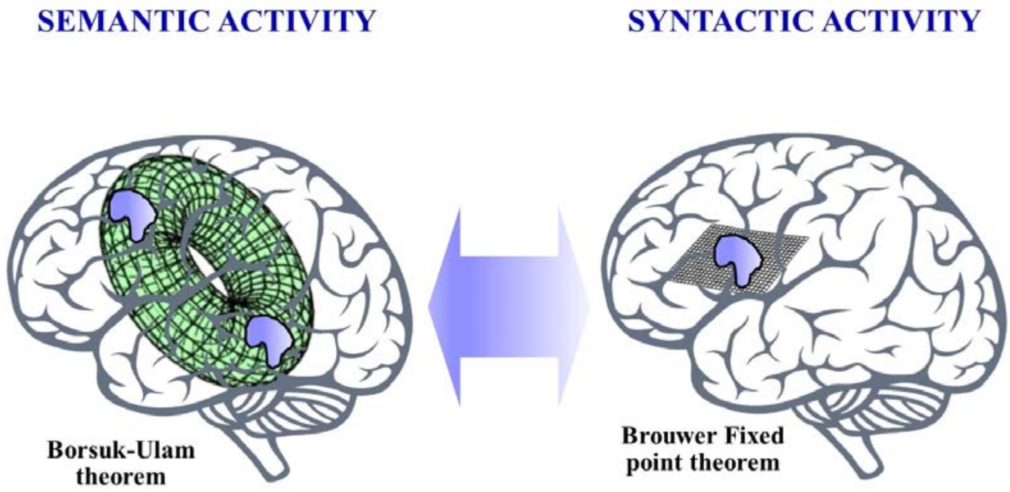


**Figure 3**. The semantic and syntactic brain activity in a topological framework. Mappings take place between brain regions which are temporarily equipped with different functional dimensions. On the left, the strBUT dictates that higher dimensional activities lie on a torus, where scattered antipodal strings stand for the cortical areas activated during semantic activity. On the right, syntactic processes lie on a 2D structure. In this latter case, the movements of the more localized strings contained in the Broca's area are dictated by the FPT. Note that the process is fully reversible, depending on the direction of the continuous mapping from one dimension to another.

## A TOPOLOGICAL MODEL OF KNOWLEDGE ISTINCT

The striking correspondence between the first Wittgenstein's and Perlovsky's approaches tells us that we can use the powerful tool of the fuzzy logic for the syntactic/semantic assessment of the human language. Indeed, Wittgenstein emphasizes the correspondence between the world and the syntactic language, while Perlovsky analyzes the relationships between the world and human semantic perception. Putting together the two issues, we achieve a useful framework: Perlovsky's computations can be used in order to correlate the world and the two components of the human language. Here follows the procedure in order to operationalize the anticipated blend between the two approaches. We will show how, for example, in Tractatus' terms, primary prepositions might stand for higher-dimensional sets of the lower-dimensional words. In particular, as an example, we will focus on the computational treatment of one of the tenets of Perlovsky's approach, *i.e.*, what he calls "knowledge instinct".

**Knowledge instinct and topological mappings**.

The derivation of knowledge is represented concisely as a fibre bundle. Briefly, a ***fibre bundle*** is a triple $(E,\pi,B)$, where $\pi : E \to B$ is a projection mapping from a bundle space E to a base space B (Husemoller, 1994). Fibre bundles are on the threshold of an operational view of a complex collection of mappings that includes a projection mapping. This is the case, since it is a straightforward task to extract from a fibre bundle the steps of an algorithm (aka, precise prescription leading to implementations in different settings). The derivation of knowledge instinct L is quite complex. So, a concise view of this derivation provided by a fibre bundle is needed. A fibre bundle representation of the derivation of knowledge instinct L is given in **Figures 4** and **5**. Each arrow $\longmapsto$ in Figure 5 represents a mapping. The **blue** arrows represent ordinary mappings that carry the derivation forward. The **blue** arrow represents a projection mapping from X(n) to the brain, which results in a gathering of antipodal evaluations of X(n), namely, $m\left(\varphi(X(n)),\varphi(-X(n))\right)$. A particular value of knowledge instinct $L(M,X)$ results from a synthesis of two signals: $X(n)$ and $M\left(Sm, m\left(\varphi(X(n)),\varphi(-X(n))\right)\right)$.

Let us note that, by means of the above formal expressions, we are allowed to produce a preliminary synthesis on a realistic topological, multi-level fuzzy logic that accommodates the basics developments for a unified mathematical treatment of both syntax and semantics. Finally, it is a deep semantics where all instances of cognitive and emotional complexity may be included.

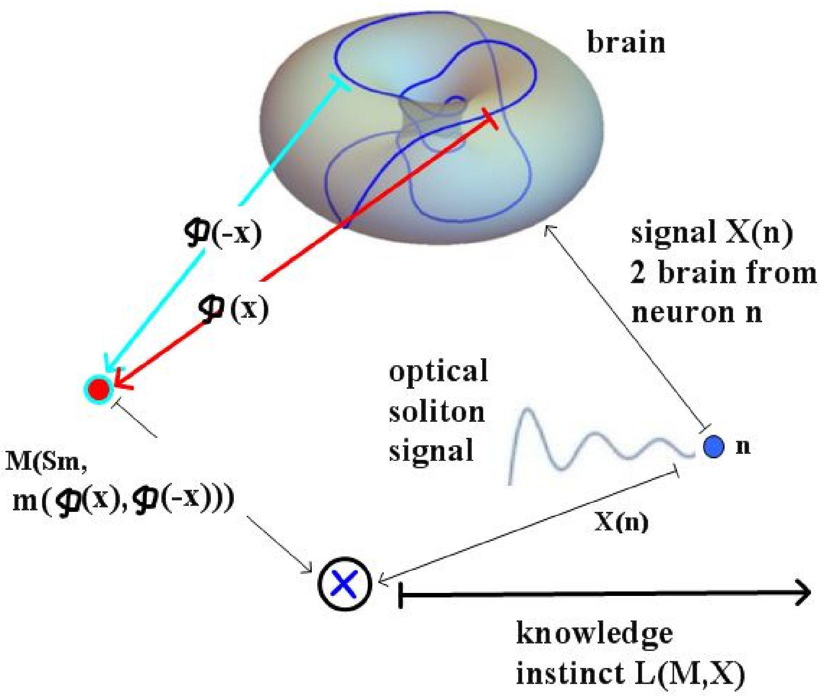


**Figure 4.** Knowledge instinct L(M, X) is a cross product of mappings:

$X(n) \otimes M\left(Sm, m\left(\varphi(X(n)), \varphi(-X(n))\right)\right) \rightarrow L(M, X)$, where $X(n)$ is the Gaussian of a signal impacting on neuron *n* and concept model $M\left(Sm, m\left(\varphi(X(n)), \varphi(-X(n))\right)\right)$ that includes input from cortical antipodal evaluations of X(n) from a toroidal view of the inner workings of the brain, *i.e.*, antipodal values $\varphi(X(n)), \varphi(-X(n))$ originate a torus-shaped region of the brain. The cross product is mapped to knowledge instinct $L(M, X)$.

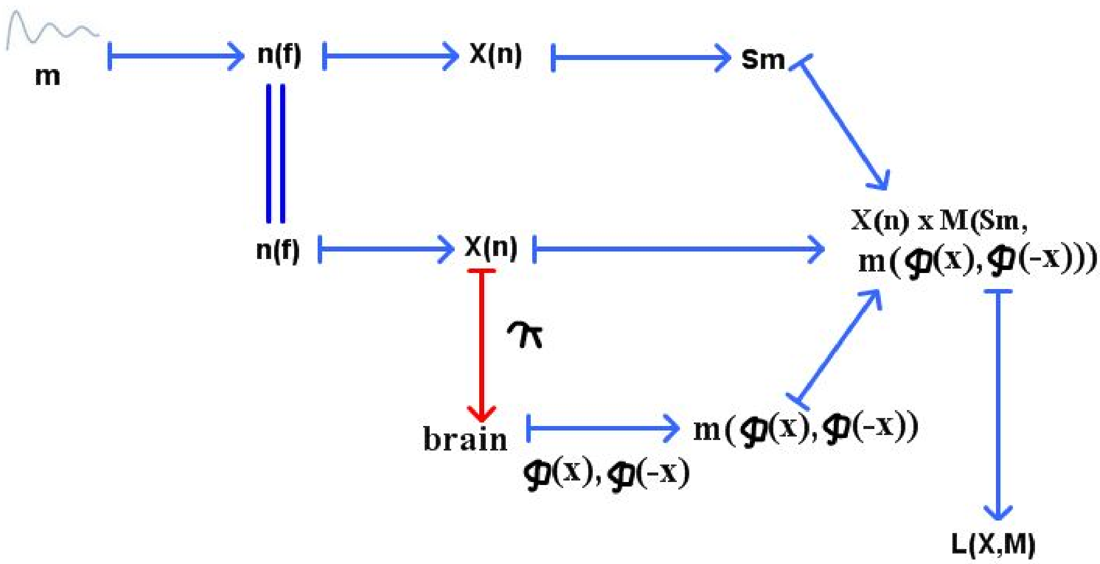


**Figure 5.** Fibre bundle representation of knowledge instinct acquisition.

## CONCLUDING REMARKS

Starting from Wittgenstein's and Perlovsky's accounts, we have shown how semantic and syntactic abilities of the brain could be investigated in terms of algebraic topology. We also illustrated how the syntactic content of the Tractatus can be assessed in terms of the current neuroscientific language. Our review of the literature data reveals how the connectivity among the brain-active centers changes during the presentation of syntactic and semantic inputs (see Benedetti et al., 2010; Fingelkurts et al., 2003 among others), where semantic arguments are based on meaning for a truth function, while syntactic propositions on symbolic correctness and completeness. Further, we described how to use the powerful tools of fuzzy logic and topology for the systematic evaluation of human language and syntactic/semantics assessment. Building upon Perlovsky's computations blended with topological dynamic approaches, a new avenue appears in order to genuinely correlate the world and the two intrinsic components of human language – the syntactic/semantic multidimensionality of human words.

It is noteworthy that these findings also prepare for a novel computational and logical approach beyond brain functions. Despite computers can process input on a 2D surface, they are not yet able to evaluate the truth function, based on meaning as True or False. Although massively parallel computers are available (Angel and Leong, 2014), their kind of multidimensionality does not tackle the outstanding problem of semantics.

Indeed, the current multidimensional efforts in computer science focus on: either access methods (sets of multidimensional points giving computers support at the physical level (Gaede and Günther, 1998)), or digital signal processing (Pastizzo et al, 2002), or data sampling (Mersereau and Speake, 1983). Therefore, multiple dimensions are currently used merely as representational devices for data analysis, causing a limitation in the potential of studies assessing artificial intelligence. The fact that brain nodes can be embedded in higher dimensional spaces suggests the possibility of manufacturing computers equipped with multidimensional nodes endowed in 3D geometric spaces. Yet a computer can be hypothesized having connections that are topologically 4D, though embedded in a simple 3D space. The latter computers, implemented in ordinary 3D space, but equipped with nodes with the same number of neighbors as points in a 4D cube, could have the capability to perform semantic operations. Complex semantics can be extracted from the statistics of input features using attractor neural networks (Ursino et al., 2009, 2010, 2011, 2014). These Authors focused on how similarity among objects, feature dominance and distinctiveness can be naturally encoded using a "simple" Hebbian training. Their model includes a lexical network, representing word-forms and a semantic network composed of several areas, each one coding for a different feature. They use a simple taxonomy of schematic objects, i.e., a vector of features, with shared categories and distinctive features (individual members). However, their model differs from ours, because what they address is the contra-hypothesis to ours, namely, that semantic meaning is represented in memory as a collection of features that *shrinks* and compacts the sum of previous sensory, motor, and emotional experiences.

## A PHILOSOPHICAL CODA

Our logical/topological approach embodies several previous suggestions by various authors. Hilbert's program states that every true proposition must be demonstrated starting from one of the available axioms. Except for those axioms, no other propositions are considered to have inherent truth value, rather they must necessarily be assessed through algorithms and computational processing. This means that a demonstrable proposition is syntactic, because it could be derived axiomatically by inspection of the symbols, independent on semantic meaning. However, the Gödel's first incompleteness theorem suggests that, given a group of arithmetical axioms, there will always be a true arithmetic to be discovered which cannot be demonstrated starting from the axioms suggested by Hilbert's approach. Indeed, the theorem states that, if we limit ourselves to syntactic methods of reasoning, then there will always be truths that are not accessible to us.

Syntactic models are inadequate to understand all the properties of an otherwise semantically understandable model. For example, our mental model of natural numbers cannot be completely characterized through syntactic methods. Further, how can we be sure that all the axioms are true? Their truth depends just on the universe of discourse, because the concept of truth has to do with semantics. According to formalists, axioms are indeed just starting points of linguistic jokes (Wittgenstein, 1953), so that the choice of an axiom looks like writing the rules of a table game. Therefore, every proposition, even if entirely false, is theoretically demonstrable, provided we start from an erratic group of faulty axioms.

Our model sheds new light on some of these issues. While Hilbert describes real objects in terms of mathematical axioms, we describe them in terms of projections and mappings. Maps from the lower syntactic dimension to the higher semantic dimension rule out the limitations invoked by Gödel's theorems. In a language, paraphrasing Charles Ball (Ferguson, Farwell, 1975), the groups of words and expressions are correlated both with their shapes and meanings, which pertain to the same conceptual sphere and to different dimensions. We could imagine that syntax works with individual symbols, while semantics works instead with groups of intrinsically correlated symbols. We mention in passing that we did not tackle the issue of the third linguistic subfield (together with syntax and semantics) of *pragmatics,* which investigates the ways in which context and use contribute to meaning (Mey 2001). Our results form a platform model for a forthcoming interpretation of pragmatics, in terms of dimensions even higher than the semantic ones.

We conclude with excerpts from Wittgenstein's Philosophical Investigations (1953), where the author seems to foreshadow the topological computational model framework in which propositions are embedded:

> "*Suppose someone said: every familiar word, in a book for example, actually carries an atmosphere with it in our minds, a 'corona' of lightly indicated uses. Just as if each figure in a painting were surrounded by delicate shadowy drawings of scenes, as it were in another dimension, and in them we saw the figures in different contexts* ($II_{VI}$)."
>
> "*But how is it possible to see an object according to an interpretation?—The question represents it as a queer fact; as if something were being forced into a form it did not really fit. But no squeezing, no forcing took place here. When it looks as if there were no room for such a form between other ones you have to look for it in another dimension. If there is no room here, there is room in another dimension* ($II_{XI}$)."

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